Fiscal Stimulus in Expectations-Driven Liquidity Traps*

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Abstract

I study expectations-driven liquidity traps in a model were agents have finite planning horizons and heterogeneous expectations. Backward-looking agents base their expectations on past observations, while forward-looking agents use model equations within their planning horizon to make forecasts. Expectations-driven liquidity traps arise when a wave of pessimism arises after a single, non-persistent, negative shock. I find that fiscal stimulus in the form of an increase in government spending or a cut in consumption taxes can be very effective in mitigating the liquidity trap. In contrast, labor tax cuts are deflationary and not an effective tool in a liquidity trap.

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1 Introduction

A large body of theoretical research has documented the state-dependence of fiscal multipliers. Christiano et al. (2009), Eggertsson (2011) and Woodford (2011) discuss why government spending multipliers are larger when the zero lower bound (ZLB) is binding than otherwise. Erceg and Lindé (2014) find that higher government spending can be used to shorten liquidity traps and even resolve them immediately if the stimulus is large enough. By contrast, lowering labor taxes is less effective, because this measure not just increases output, but also increases labor supply, implying lower wages. The resulting decrease in marginal costs for firms puts downward pressure on inflation, possibly increasing the severity and the duration of the liquidity trap.

So far, the majority of these studies have investigated the effectiveness of fiscal and monetary policy at the ZLB under the assumption of rational expectations, and in the case of liquidity traps that are purely driven by fundamental shocks, i.e., shocks that reduce the natural rate of interest. An exception is Mertens and Ravn (2014), who, also within a rational expectation framework, investigate the occurrence of liquidity traps due to the coordination of expectations on a sunspot shock.

However, a growing strain of the literature has shown the importance of bounded rationality for macroeconomic policy, especially at the ZLB (see a.o. Williams, 2006, Akerlof and Shiller, 2010, De Grauwe, 2012 and Gabaix, 2016). In this paper, I therefore study the emergence of liquidity traps due to boundedly rational and heterogeneous expectations. In line with e.g. Orphanides and Williams (2005, 2007), Milani (2007), Slobodyan and Wouters (2012) and Hommes and Zhu (2014), I find that the presence of backward-looking agents adds persistence. Therefore, long-lasting liquidity traps can arise, that are purely driven by expectations. In fact, the persistence amplification of backward-looking agents can be so severe under the zero lower bound that the economy ends up in a deflationary spiral. I therefore add a second intuitive layer of bounded rationality: finite planning horizons. This facilitates long lasting liquidity traps that do not end in a deflationary spiral. In this framework, I study the effectiveness of different measures of fiscal stimulus in mitigating expectations-driven liquidity traps.

More specifically, instead of being able to form expectations up to an infinite horizon as is usually assumed, agents in my model are relatively short-sighted and are able to plan ahead and form expectations only up to T periods into the future, as in Lustenhouwer and Mavromatis (2017) and Woodford (2018).[1] Moreover, only a fraction of agents in the modeled economy form expectation in a forward-looking manner, using their knowledge of the model equations. The other fraction of agents use a backward-looking rule of thumb, according to which all variables will mean-revert back to their steady state in the future. Such rule of thumb behavior, used by e.g. Branch and McGough (2009, 2010) and Gasteiger (2014, 2017), who label it adaptive expectations, is found to be consistent with expectations of human subjects in laboratory experiments (see e.g. Assenza et al. (2014) and Pfajfar and Zakelj, 2011) as well as with survey data (see e.g. Branch (2004, 2007)). Other works with similar heterogeneous expectations frameworks include Elton et al. (2017), Massaro (2013) and Deák et al. (2017). To the best of my knowledge, this paper is the first to combine heterogeneity in expectations with a micro-founded framework of finite planning horizons.

In this setup, I show how liquidity traps can occur due to the boundedly rational expectations of agents, and compare this scenario with a liquidity trap driven by a persistent fundamental shock. Interestingly, I find that when agents have planning horizons that are not too long, expectations-driven liquidity traps of considerable duration can arise, from which the economy eventually recovers. If, on the other hand, agents have a long or infinite planning horizon, the economy runs the risk of falling into a deflationary spiral from which

[1] See also Branch et al. (2013) for a related approach.

it does not recover without policy intervention.

I then turn to the question whether fiscal stimulus in the form of a spending increase or tax-cut can mitigate the severity and duration of expectations-driven liquidity traps and prevent deflationary spirals. In line with the literature on liquidity traps driven by fundamentals, I find that an increase in government spending is effective in mitigating a liquidity trap, while cutting labor taxes is deflationary and does not have positive effects. In addition, I also consider cutting consumption taxes as a tool for fiscal stimulus. Unlike a cut in labor taxes, I find that a cut in consumption taxes is inflationary and can considerably reduce the duration of an expectations-driven liquidity trap. This is in line with the results of Eggertsson (2011), Coenen et al. (2012) and Correia et al. (2013) in a liquidity trap driven by fundamentals. Moreover, I find that both increases in government spending and cuts in consumption taxes can prevent deflationary spirals that arise for large planning horizons, while cuts in labor taxes cannot. Finally, I find that the required size of fiscal stimulus to prevent a deflationary spiral can considerably be reduced if the central bank has a higher inflation target.

My findings regarding the effects of increases in government spending in a liquidity trap with bounded rationality are in line with those of Hommes et al. (2015) and a series of papers by Evans and coauthors. Hommes et al. (2015) conduct a laboratory experiment, where, rather than making any assumptions on agent's expectations, they let expectations be formed by human subjects in the laboratory. Without fiscal intervention, deflationary spirals regularly occur in their experiment. However, in treatments where there is a fiscal switching rule and government spending is increased when inflation is below some threshold, deflationary spirals are always prevented. This confirms the results of Evans et al. (2008), Evans and Honkapohja (2009) and Benhabib et al. (2014), who find that under adaptive learning such spending increases can always prevent deflationary spirals that would otherwise have arisen because of the zero lower bound.

However, my results are in sharp contrast with those of Mertens and Ravn (2014), who find that, contrary to widely held views, in their sunspot equilibria, increasing government spending at the zero lower bound is deflationary and is not very effective in mitigating a liquidity trap, while cutting labor taxes is inflationary and does mitigate the liquidity trap considerably. The current paper shows that this reversal of traditional results is not a general feature of liquidity traps driven by expectations, but depends crucially on the choice of modeling an expectations-driven liquidity trap as a sunspot equilibrium.

I furthermore add to the above literature by not only considering government spending increases (and in case of Mertens and Ravn (2014) also labor tax cuts) as a tool for fiscal stimulus, but also studying the effect of consumption tax cuts in an expectations-driven liquidity trap. To the best of my knowledge this paper is the first to do so in a framework of bounded rationality.

The remainder of the paper is organized as follows. In Section 2, the model and expectation formation processes are outlined. In Section 3, I present how expectations-driven liquidity traps can occur, and how their duration depends on the behavioral features of the model. In Section 4, the effectiveness of different fiscal stimulus packages is investigated. Finally, Section 5 concludes.

2 Model

The model is made up by a continuum of households $i \in [0,1]$, a continuum of firms $j \in [0,1]$ and a monetary and fiscal authority. Moreover, there are two types of households and firms. A fraction α of households and firms forms expectations in a backward-looking manner and a fraction $1-\alpha$ forms expectations in a forward-looking manner. The expectations of these two types of households and firms will be specified in Section 2.5.

2.1 Households

Households want to maximize their discounted utility over their planning horizon (T periods), and they also value the state they expect to end up in at the end of these T periods (their state in period T+1). They are not able to rationally induce (by solving the model forward), how exactly they should value their state in period T+1. Instead households use a rule of thumb to evaluate the value of their state (their wealth). As in Lustenhouwer and Mavromatis (2017) and Woodford (2018), their objective function therefore exists of a sum of utilities, $U(\cdot)$, out of consumption and leisure for the periods within their horizon, as well as an extra term with a function $V(\cdot)$ that is increasing in end of horizon wealth:

$$\max_{C^i,H^i,B^i} \tilde{E}_t^i \left[\sum_{s=t}^{t+T} \beta^{s-t} \xi_s U(C_s^i, H_s^i) + \beta^{T+1} V\left(\frac{B_{t+T+1}^i}{P_{t+T}}\right)\right], \tag{1}$$

subject to

$$(1+\tau_\tau^c)P_\tau C_\tau^i + \frac{B_{\tau+1}^i}{1+i_\tau} \leq (1-\tau_\tau^l)W_\tau H_\tau^i + B_\tau^i + P_\tau \Xi_\tau - P_\tau LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{2}$$

Here B_t^i are nominal bond holdings from household i at the beginning of period t; C_τ^i and H_τ^i are the household's consumption and labor; Ξ_t are real profits from firms which are equally distributed among households; τ_τ^c and τ_τ^l are respectively the consumption tax and labor tax rates; LS_τ denotes lump sum taxes; i_τ is the nominal interest rate; P_τ is the price level; and W_τ is the nominal wage rate. Finally β is the household's discount factor, while ξ_τ is an exogenous preference shock.

Dividing the budget constraint by P_τ gives

$$(1+\tau_\tau^c)C_\tau^i + \frac{B_{\tau+1}^i}{(1+i_\tau)P_\tau} \leq (1-\tau_\tau^l)w_\tau H_\tau^i + \frac{B_\tau^i}{P_\tau} + \Xi_\tau - LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{3}$$

It is assumed that households have CRRA preferences for consumption and labor, so that

$$U(C_s^i, H_s^i) = \frac{(C_s^i)^{1-\sigma}}{1-\sigma} - \frac{(H_s^i)^{1+\eta}}{1+\eta}. \tag{4}$$

Moreover, the functional form of $V(.)$ is given by

$$V(x) = \frac{1}{1-\beta}\left[\frac{1}{1-\sigma}\left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{1+\bar{\tau}^c}\frac{x}{\bar{\Pi}}\right)^{1-\sigma}\right], \tag{5}$$

with $\Lambda = (1-\bar{\tau}^l)\bar{w}\bar{H} + \bar{\Xi}$ equal to steady state net income.

Equation (5) is (dropping terms independent of x) the continuation value that solves the Bellman equation

$$V(x) = \max_c\{U(C, \bar{H}) + \beta V(x')\}, \quad s.t. \quad x' = \frac{\bar{\Pi}}{\beta}\left[(1-\bar{\tau}^l)\bar{w}\bar{H} + \frac{x}{\bar{\Pi}} + \bar{\Xi} - \bar{LS} - (1+\bar{\tau}^c)C\right]. \tag{6}$$

Similar to Woodford (2018), this optimization problem gives the optimal intertemporal consumption decision of households assuming that taxes, wages, hours worked, inflation, interest rates and profits are all *in steady state*. That is, the only variables that are allowed to vary under this optimization problem are consumption (C) and debt ($x = \frac{B^i_{t+T+1}}{P_{t+T}}$).

Under this way of deriving (5), agents are not sophisticated enough to plan how their hours worked, wages and aggregate variables like inflation, interest rates and profits would change after their horizon if they would vary their consumption plan after their horizon.

However, using this Value function in Equation (1), households make fully optimal decisions in steady state. Moreover, $V(x)$ is increasing in x. Therefore, agents realize that holding more bonds at the end of their horizon will result in more utility. The value function hence captures partly how future utility depends on end of horizon wealth, but in a boundedly rational manner that only approximates the true value function.

The first order conditions of the maximization problem (1) subject to (3) are

$$\xi_\tau (C^i_\tau)^{-\sigma} = \lambda^i_\tau (1+\tau^c_\tau), \qquad \tau = t, t+1, ...t+T, \tag{7}$$

$$\xi_\tau (H^i_\tau)^{\eta} = \lambda^i_\tau (1-\tau^l_\tau) w_\tau, \qquad \tau = t, t+1, ...t+T, \tag{8}$$

$$\lambda^i_\tau = \beta \tilde{E}^i_t \frac{(1+i_\tau)\lambda^i_{\tau+1}}{\Pi_{\tau+1}}, \qquad \tau = t, t+1, ...t+T-1, \tag{9}$$

$$\lambda^i_{t+T} = \beta(1+i_{t+T}) \frac{1}{\bar{\Pi}(1+\bar{\tau}^c)} \left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{(1+\bar{\tau}^c)\bar{\Pi}} \frac{B^i_{t+T+1}}{P_{t+T}} \right)^{-\sigma}, \tag{10}$$

Next we define a measure of real bond holdings, scaled by steady state output: $b_t = \frac{B_t}{P_{t-1}\bar{Y}}$. Substituting for this expression in (10) and (3) gives

$$\lambda^i_{t+T} = \beta(1+i_{t+T}) \frac{1}{\bar{\Pi}(1+\bar{\tau}^c)} \left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{(1+\bar{\tau}^c)} \frac{\bar{Y} b^i_{t+T+1}}{\bar{\Pi}} \right)^{-\sigma}, \tag{11}$$

and

$$(1+\tau^c_\tau) C^i_\tau + \bar{Y} \frac{b^i_{\tau+1}}{1+i_\tau} \leq (1-\tau^l_t) w_\tau H^i_\tau + \frac{\bar{Y} b^i_\tau}{\Pi_\tau} + \Xi_\tau - LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{12}$$

2.2 Firms

There is a continuum of firms producing the final differentiated goods. Each firm has a linear technology with labor as its only input

$$Y_t(j) = H_t(j). \tag{13}$$

There is monopolistic competition and it is assumed that in each period a fraction $(1-\omega)$ of firms can change their price, as in Calvo (1983).

Each firm is run by a household and follows the same heuristic for prediction of future variables as that household in each period. Moreover, firms are also short sighted. That is, they will form expectations about their marginal costs and the demand for their product for T periods ahead only. However, as in the case of the household problem, firms also care about their state at the end of the horizon, and consider the possibility that they might then still be stuck with the price that they set now. The problem of firm j that can reset its price is then to maximize the discounted sum of its expected future profits within its horizon plus its perceived value of its state at the end of the horizon. In utility terms, and using the demand for good j, this can be written as

$$\tilde{E}_t^j \left(\sum_{s=0}^{T} \omega^s \beta^s \lambda_{t+s}^j \left[\left(\frac{p_t(j)}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - mc_{t+s} \left(\frac{p_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \right] + \omega^{T+1} \beta^{T+1} \tilde{V}\left(\frac{p_t(j)}{P_{t+T}}\right) \right), \tag{14}$$

where λ_t^j is the Lagrange multiplier of the utility optimization problem of the household (j) that runs firm j.

As in Woodford (2018), $\tilde{V}(\cdot)$ describes the continuation value of real profits in utility terms as a function of the relative price. As in case of the household, this value function is obtained from the assumption that all variables other than the relative price of the firm (such as output, wages and the aggregate price level) are in steady state. This value function therefore satisfies

$$\tilde{V}(r) = \bar{\lambda} \left(\left(\frac{r}{\bar{\Pi}} \right)^{1-\theta} \bar{Y} - \left(\frac{r}{\bar{\Pi}} \right)^{-\theta} \bar{Y} \bar{mc} \right) + \omega \beta \tilde{V}\left(\frac{r}{\bar{\Pi}}\right) + (1-\omega) \beta \tilde{V}^{opt}, \tag{15}$$

where $\tilde{V}^{opt}$ is next period's value for a firm that can re-optimize next period. Since $\tilde{V}^{opt}$ does not influence the current decision problem of the firm (since it is independent of r), I ignore it and let the functional form of $\tilde{V}(r)$ be

$$\tilde{V}(r) = \frac{1}{1 - \omega\beta\bar{\Pi}^{\theta-1}} \bar{\lambda} \left(\frac{r}{\bar{\Pi}} \right)^{1-\theta} \bar{Y} - \frac{1}{1 - \omega\beta\bar{\Pi}^{\theta}} \bar{\lambda} \left(\frac{r}{\bar{\Pi}} \right)^{-\theta} \bar{Y} \bar{mc}. \tag{16}$$

The first order condition for maximizing (14) with respect to $p_t(j)$ then is

$$\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\lambda_{t+s}^j}{P_{t+s}} Y_{t+s} \left[(1-\theta) \left(\frac{p_t^*(j)}{P_{t+s}} \right)^{-\theta} + \theta mc_{t+s} \left(\frac{p_t^*(j)}{P_{t+s}} \right)^{-1-\theta} \right] \tag{17}$$

$$+(\omega\beta)^{T+1} \frac{\bar{\lambda}}{\bar{\Pi} P_{t+T}} \bar{Y} \left[\frac{1-\theta}{1-\omega\beta\bar{\Pi}^{\theta-1}} \left(\frac{p_t^*(j)}{\bar{\Pi} P_{t+T}} \right)^{-\theta} + \frac{\theta \bar{mc}}{1-\omega\beta\bar{\Pi}^{\theta}} \left(\frac{p_t^*(j)}{\bar{\Pi} P_{t+T}} \right)^{-1-\theta} \right] = 0,$$

where $p_t^*(j)$ is the optimal price for firm j if it can re-optimize in period t.

Next, turn to the evolution of the aggregate price level. I assume that the set of firms that can change their price in a period is chosen independently of the type of the household running the firm, so that the distribution of expectations of firms that can change their price is identical to the distribution of expectations of all firms. Since decisions of firms only differ in so far as their expectations differ, it follows that the aggregate price level evolves as

$$P_t = [\omega P_{t-1}^{1-\theta} + (1-\omega) \int_0^1 p_t^*(j)^{1-\theta} \mathrm{d}j]^{\frac{1}{1-\theta}}. \tag{18}$$

2.3 Government and market clearing

The government issues bonds and levies labor taxes (τ_t^l), consumption taxes (τ_t^c) and lump sum taxes (LS_t) to finance its (wasteful) spending (G_t). Its budget constraint is given by

$$\frac{B_{t+1}}{1+i_t} = P_t G_t - \tau_t^l W_t H_t - \tau_t^c P_t C_t - P_t LS_t + B_t, \tag{19}$$

with $H_t = \int H_t^i di$ and $B_t = \int B_t^i di$ aggregate labor and aggregate bond holdings respectively. Dividing by $\bar{Y} P_t$ gives

$$\frac{b_{t+1}}{1+i_t} = g_t - \tau_t^l w_t \frac{H_t}{\bar{Y}} + \tau_t^c \frac{C_t}{\bar{Y}} - \frac{LS_t}{\bar{Y}} + \frac{b_t}{\Pi_t}, \tag{20}$$

where $b_t = \frac{B_t}{P_{t-1}\bar{Y}}$ and $g_t = \frac{G_t}{\bar{Y}}$ are the ratios of debt to steady state GDP and government expenditure to steady state GDP, respectively.

Market clearing is given by

$$Y_t = C_t + G_t = C_t + \bar{Y} g_t. \tag{21}$$

g_t, τ_t^l and τ_t^c can be set discretionary by the government to counteract liquidity traps. Lump sum taxes, LS_t, adjust to stabilize debt,

$$LS_t = \bar{LS} \left(\frac{b_t}{\bar{b}} \right)^{\gamma_{LS}}. \tag{22}$$

The monetary policy rule is given by

$$1 + i_t = \max \left(1, (1 + \bar{i}) \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_1} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_2} \right). \tag{23}$$

2.4 Linearized model

In Appendix B, the model is linearized around a general inflation target, and the resulting model equations are given by Equations (77) through (79) in that appendix. In most of the paper, I assume a zero inflation target, and only in Section 4.3 I focus on the effects of changing the inflation target to a positive value. When the inflation target is zero, the linearized model equations reduce to

$$
\begin{aligned}
(1-\nu_{y0})\hat{Y}_t = &\frac{1}{\rho_0}\tilde{b}_t + g_t + \nu_{\tau 0}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\tau}^l_{t+s}) + \nu_{g0}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{g}_{t+s}) + \nu_{y0}\sum_{s=1}^{T}\beta^s(\bar{E}_t\hat{Y}_{t+s}) \\
&- \mu_0\sum_{s=1}^{T}\beta^s\sum_{j=1}^{s}(\bar{E}_t i_{t+j-1} - \bar{E}_t\hat{\pi}_{t+j}) + \frac{\bar{b}}{\rho_0}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s} - \bar{E}_t\hat{\pi}_{t+s}) \\
&- \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho_0}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho_0}\bar{E}_t i_{t+T} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) \qquad (24) \\
&\mu_{\xi 0}\xi_t - \mu_0\sum_{s=1}^{T}\beta^s\bar{E}_t\xi_{t+s} + \nu_{c10}\tilde{\tau}^c_t + \nu_{c20}\sum_{s=1}^{T}\beta^s\bar{E}_t\tilde{\tau}^c_{t+s}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_t = &\kappa(\eta + \frac{\sigma}{1-\bar{g}})\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\hat{Y}_{t+s} - \frac{\kappa\sigma}{1-\bar{g}}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{g}_{t+s} \qquad (25) \\
&+ \frac{\kappa}{1+\bar{\tau}^c}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{\tau}^c_{t+s} + \frac{\kappa}{1-\bar{\tau}^l}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{\tau}^l_{t+s} + \frac{1-\omega}{\omega}\sum_{s=1}^{T}\omega^s\beta^s\bar{E}_t\hat{\pi}_{t+s},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{b}_{t+1} = &\frac{1}{\beta}\tilde{g}_t - \frac{\bar{\tau}^c}{\beta}(\hat{Y}_t - \tilde{g}_t) - \frac{1-\bar{g}}{\beta}\tilde{\tau}^c_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t) - \frac{\bar{LS}}{\beta\bar{Y}}\hat{LS}_t \qquad (26) \\
&- \frac{\bar{w}}{\beta}\left[\bar{\tau}^l\left((1+\eta+\frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l}\right) + \tilde{\tau}^l_t\right],
\end{aligned}
$$

with coefficients defined in Appendix B.4. The operator $\bar{E}_t$ denotes aggregate expectations of all agents at time t.

Linearized monetary and fiscal policy equations are given by

$$
\hat{i}_t = \phi_1\hat{\pi}_t + \phi_2\hat{Y}_t, \qquad (27)
$$

$$
\hat{LS}_t = \gamma_{LS}\tilde{b}_t. \qquad (28)
$$

2.5 Expectations

There are two types of agents in the economy: forward-looking agents and backward-looking agents. Backward-looking agents consider the last observation of all variables, and consider this observation to be most informative about the current state of the economy, and its future evolution. They however do not expect the economy to stay in its current state forever, but instead expect mean-reversion to the target steady state in the future. In particular their expectations about output s periods from now is given by

$$E_t^b \hat{Y}_{t+s} = \rho^{s+1} \hat{Y}_{t-1} \tag{29}$$

Branch and McGough (2010) and Gasteiger (2014) and others refer to these expectations as adaptive expectations. Expectations about inflation, debt, the nominal interest rate, government spending, taxes and (in case of a positive inflation target) price dispersion are formed analogously.

The main results of the paper also qualitatively hold if it is instead assumed that backward-looking agents use an adaptive learning algorithm with constant gain, as in, e.g., Benhabib et al. (2014). This is discussed in Appendix D.3.

As will be discussed below, I allow for a shock to output and inflation expectations of backward-looking agents. These then become

$$E_t^b \hat{Y}_{t+s} = \rho^{s+1} (\hat{Y}_{t-1} + expshock_t) \tag{30}$$

$$E_t^b \hat{\pi}_{t+s} = \rho^{s+1} (\hat{\pi}_{t-1} + expshock_t) \tag{31}$$

Forward-looking agents understand the mechanics of the economy. They also know what fraction of agents in the economy is backward-looking and what fraction is forward-looking. Moreover, they observe the expectations that backward-looking agents have formed in the current period, about the next T periods. They are however not rational enough to know exactly how backward-looking agents are forming their expectation. Therefore, forward-looking agents have no reason to believe that backward-looking agents will later revise currently formed expectations about a particular future period. Instead, forward-looking agents assume that backward-looking agents will stick with the expectations about future periods that they have currently formed. Mathematically, this implies $E_t^f[E_{t+k}^b Y_{t+s}] = E_t^b Y_{t+s}$, with $k < s \leq T$.

Moreover, forward-looking agents are restricted in their ability to sophisticatedly think about the future by their planning horizon. As in Lustenhouwer and Mavromatis (2017), I assume that forward-looking agents rationally use the model equations within their horizon to form expectations, but that they are not able to form expectations for variables outside their horizon in a sophisticated manner.

Because of the above mentioned bounded rationality, forward-looking agents know the model equations and the structural form of the minimum state variable solution of the model, but their computed solution will typically deviate somewhat from fully rational expectations. The precise algorithm that forward-looking agents use to compute expectations is described in Appendix C.

3 Liquidity traps

In this section, I show that in the above model with forward-looking and backward-looking agents, liquidity traps can arise that are driven by expectations rather than by fundamentals. In order to highlight the role of expectations, I initiate a liquidity trap by a non-persistent shock directly to both output and inflation expectations. In Appendix D.1 it is shown that such a liquidity trap could also arise following a single, non-persistent shock to the fundamentals of the economy. In that case, the subsequent fall in output and inflation expectations is the result of agents observing low output and inflation. In the main body of the paper however, I keep the analysis more general. The initial fall in expectations could then also be though to follow something outside the model such as a global panic, or a financial crash.

The intuition for an expectations-driven liquidity trap of multiple periods to arise in the behavioral model, even after a non-persistent shock to expectations, is the following. Becasue of low output and inflation expectations, agents reduce consumption and prices, so that output and inflation are low. This reinforces the low expectations of backward-looking agents and the liquidity trap continues.

This case of a liquidity trap driven by expectations of backward-looking agents is compared with a liquidity trap driven by fundamentals, where there is a persistent negative preference shock that creates a desire to save. Similar shocks are used to model a liquidity trap by e.g. Eggertsson (2011) and Mertens and Ravn (2014).

Below, I illustrate how liquidity traps can arise (Section 3.2), and how the occurrence of expectations-driven liquidity traps depends on the size of the shock and the fraction of backward-looking agents (Section 3.3). In addition, there is an important role for the planning horizon of agents. This will be the focus of Section 3.4. First, the parameterization is discussed in Section 3.1.

3.1 Parameterization

In the model, one period corresponds to one quarter. I set the discount factor to $\beta = 0.99$, the coefficient of relative risk aversion to $\sigma = 1$, the inverse of the Frisch elasticity of labor supply to $\eta = 2$, the elasticity of substitution to $\theta = 6$ and the Calvo parameter to $\omega = 0.75$. These values are relatively standard in the literature. Steady state fiscal variables are chosen more or less in line with US historical averages as follows: steady state government spending as a share of GDP is set to $\bar{g} = \bar{G}/\bar{Y} = 0.3$; the steady state labor and consumption tax rate are set to respectively $\bar{\tau}^l = 0.2$ and $\bar{\tau}^c = 0.08$, and steady state lump sum taxes are set to $\bar{LS} = 0.08$. Monetary and fiscal policy variables are set to $\phi_1 = 1.5$ and $\phi_2 = 0.157$ (implying a response to output of around 0.6 when annual data are used) and $\gamma_{LS} = 1$. I further set the mean reversion in the expectations of backward-looking agents to 0.8. With that calibration, backward-looking agents expect the deviation of variables from steady state to have reduced to one tenth of the current deviation after approximately 10 quarters. In Appendix D.2, I study robustness to the calibration of this parameter. Generally, I consider different values of the planning horizon T, with some illustrations being performed under the assumption of $T = 8$.

3.2 Illustration of expectations-driven liquidity traps

Figure 1 plots the simulated time series of output, inflation, the expectations of backward-looking agents about output and inflation, the nominal interest rate and the preference shock in case of $T = 8$. The dashed green curves depict the case of a liquidity trap driven by fundamentals. Here all agents are forward-looking, and there is a negative preference shock in period 2, with auto-correlation coefficient 0.75. Because the fundamentals remain low for a number of periods, agents in the economy choose relatively low consumption and prices for several periods. As a result, the liquidity trap lasts up to and including period 8.

The purple curves depict a liquidity trap driven by expectations, where 50% of agents in the economy are backward-looking. The blue dotted curves in the middle panels represent expectations paths of backward looking agents that they hold at different points in time. The purple curves that connect these expectation paths represents the evolution of their expectations about one period in the future.

In period 2, there is a shock to output and inflation expectations of backward-looking agents which reduces there expectations about period 3 to 10 (see drop in period 3 in middle

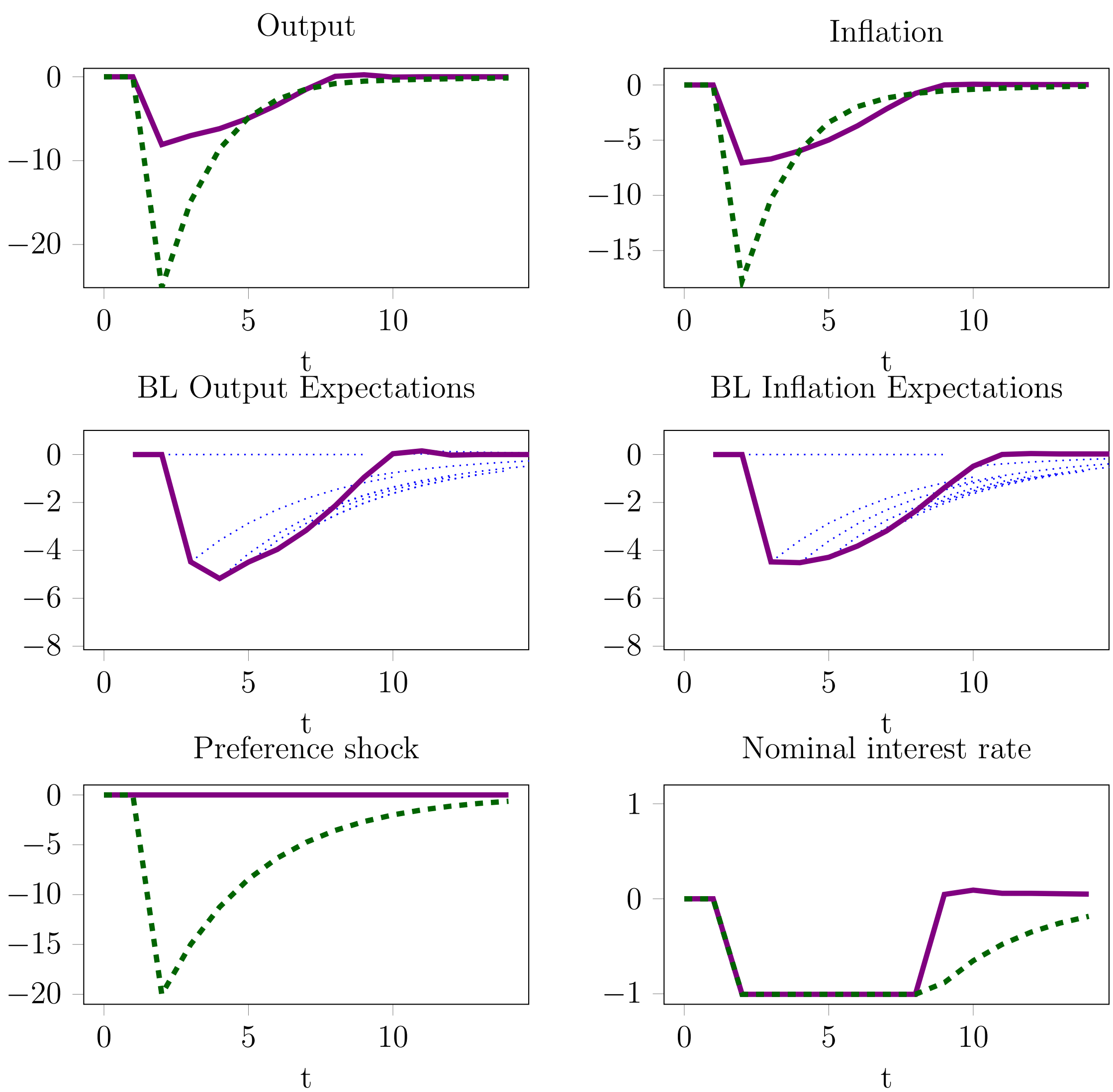


Figure 1: Liquidity traps for $T = 8$. The solid purple curves depict the case of an expectations-driven liquidity trap, while the dashed green curves depict the case of a liqudity trap driven by a persistent negative preference shock with only forward-looking agents. The blue dotted curves in the middle panels represent expectation paths of backward looking agents that they hold at different points in time. The purple curves that connect these expectation paths represents the evolution of their expectations about one period in the future.

panels). The size of the shock is $expshock_t = -0.07$, which is equivalent to a reduction of last periods output and inflation of 7% (see equations (30) and (31)). Because of their low expectations, backward-looking agents reduce consumption and prices in period 2, which leads to a binding zero lower bound on the nominal interest rate. Forward-looking agents are aware of this, and rationally expect lower inflation and output as well. In the next period, backward-looking agents observe low output and inflation so that their expectations remain low (middle panels), resulting again in low output and inflation and a binding zero lower bound. This expectations-driven liquidity trap lasts up to and including period 8 (a total of 7 quarters), just as the liquidity trap caused by a persistent fundamental shock.

3.3 Duration of liquidity trap

Next, let us move to a more robust analysis of the model dynamics and the length of the expectations-driven liquidity trap for different fractions of backward-looking agents and for different sizes of the shock to expectations.

Panel (b) of Figure 2 displays the duration of the expectations-driven liquidity trap in case of a planning horizon of $T = 8$. On the horizontal axis is the fraction of backward-looking agents and on the vertical axis is the size of the one-time negative shock to expectations (in absolute value). The figure shows the following. When the shock is small, or when the fraction of backward-looking agents is small, the liquidity trap is immediately over after the fundamental shock is over (duration of 1 period). This is the lightest region in the bottom left of the figure. For larger fractions of backward-looking agents and/or larger shocks, the liquidity trap lasts longer and longer, as can be seen from the changing color shades.

Remember that the liquidity trap with duration 7 presented in purple in Figure 1 had a share of 0.5 backward looking agents and a shock size of 7%. In panel (b) of Figure 2 it can be seen that liquidity traps of longer duration arise when the negative shock to

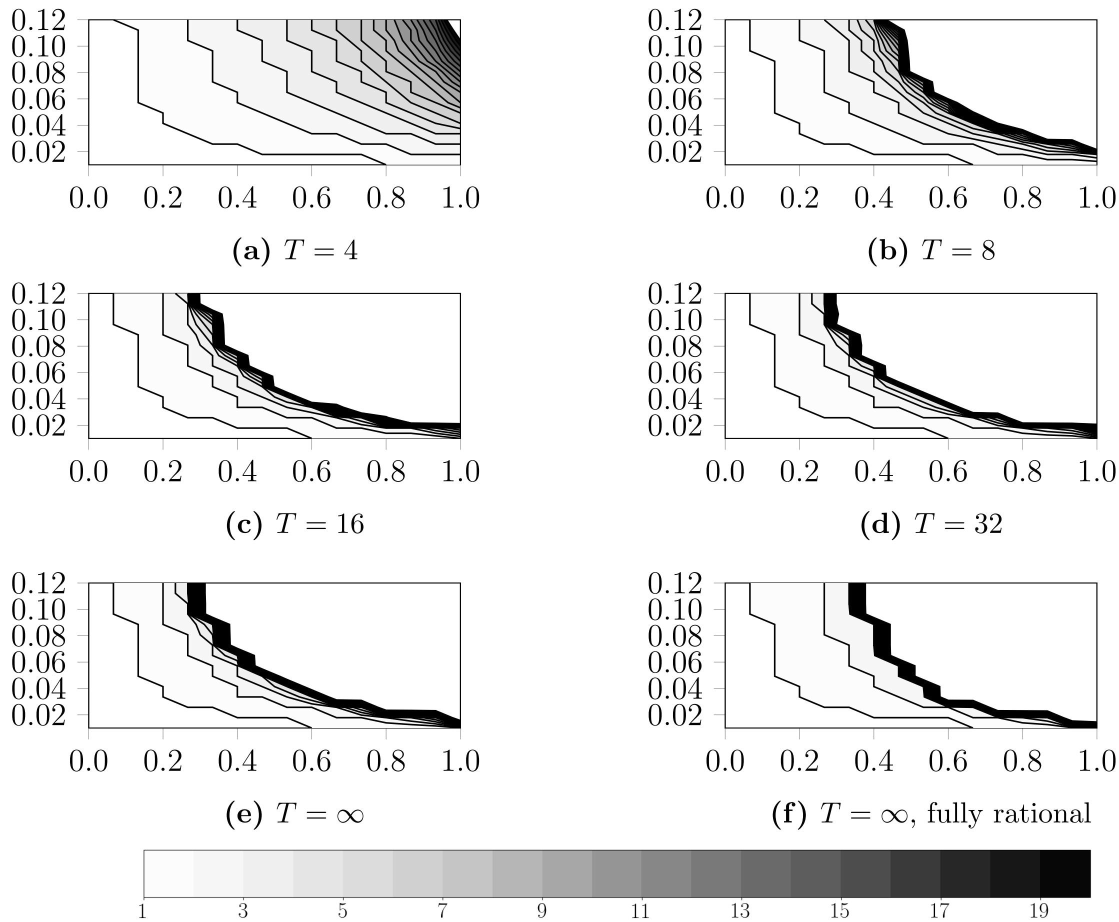


Figure 2: Length of liquidity trap for different fractions of backward-looking agents (x-axis) and different sizes of the (non-persistent) negative shock to expectations (y-axis). The different panels correspond to different planning horizons. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top right of the bottom 4 panels indicate liquidity traps of infinite length with ever decreasing inflation and output (deflationary spirals).

expectations is of larger magnitude, but especially when the fraction of backward-looking agents in the economy is made even higher.

However, for very large fractions of backward-looking agents and larger shock sizes, it is no longer the case that very long liquidity traps arise from which the economy eventually recovers. Instead, the economy falls in a deflationary spiral, with ever decreasing inflation and output. Deflationary spirals with ever lasting liquidity traps are indicated by white areas. The intuition for such a deflationary spiral to arise is that backward-looking agents expect low inflation, high real interest rates and low output for most of the periods within their horizon, and hence reduce prices and consumption with considerable magnitude. When there are enough backward-looking agents in the economy, the resulting drop in inflation and output is enough to make agents even more pessimistic in the next period, causing inflation and output to keep falling further and further.

3.4 Planning horizon

Now, consider the effect of the planning horizon of agents on the liquidity trap. Figure 3 plots in purple the expectations-driven liquidity of Figure 1, where the planning horizon equals 8 and the fraction of backward-looking agents is 50%. The dashed green curves in Figure 3 depict the case of a shorter horizon of $T = 4$, while the dashed-dotted black curves depict the case of a longer horizon of $T = 16$.

The initial shock to expectations is the same in all three cases. Nonetheless, the pessimistic expectations for $T = 8$ imply a prolonged expectations-driven liquidity trap with slow convergence to the steady state, while similar expectations for $T = 4$ imply a swift recovery, and for $T = 16$ imply a rapid drop in output and inflation and a subsequent deflationary spiral. The reason for this is the effects that pessimistic expectations about the future have on agents' consumption, labor and pricing decisions. Short sighted agents only consider a small number of future periods when making decisions. For $T = 4$, backward-

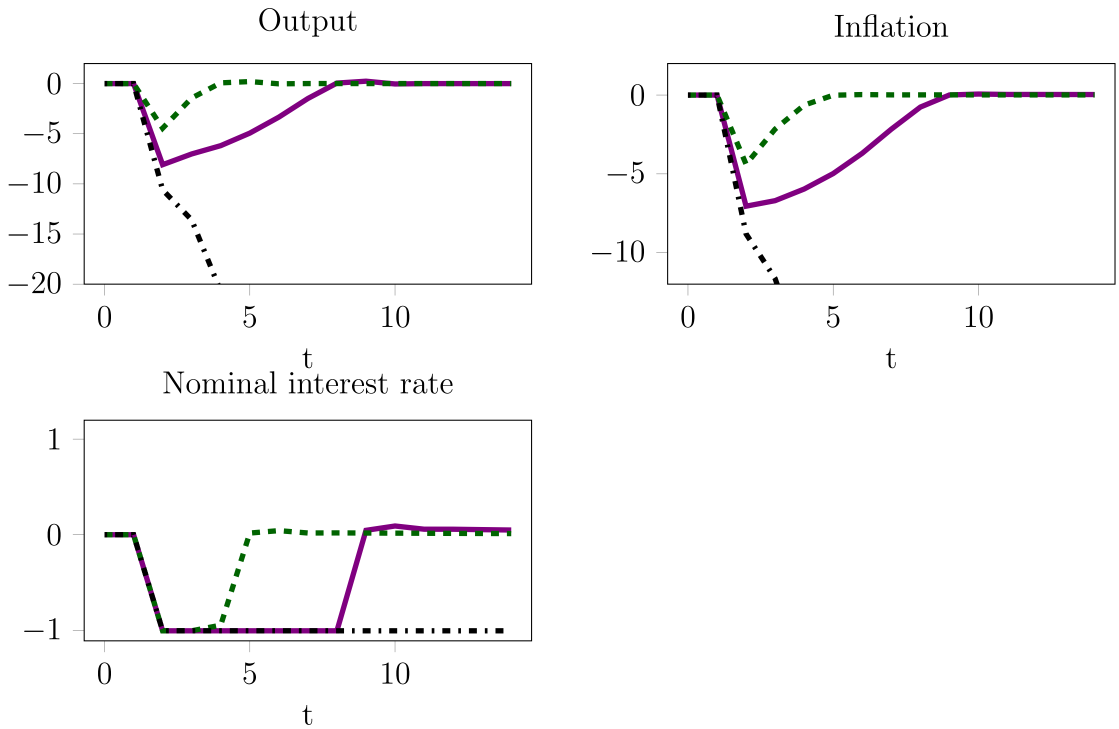


Figure 3: Expectations-driven liquidity trap for $T = 4$ (dashed green), $T = 8$ (purple) and $T = 16$ (dashed-dotted black).

looking consumers expect increased real interest rates and lowered output for the next 4 periods, and reduce their consumption somewhat accordingly. However, they do not consider what might happen after these four periods when making their decisions. An agent with a planning horizon of $T = 16$ on the other hand, plans for considerably more future periods. When this agent expects real interest rates and output to be low also five periods from now and later, he will reduce his consumption more than an agent that looks only 4 periods into the future.[2]

Next, consider more generally what the change in planning horizon implies for different values of the shock size and different values of the fraction of backward-looking agents in case of a non-persistent shock to expectations. Panel (a) of Figure 2 reproduces panel (b), but now with a planning horizon of $T = 4$ instead of $T = 8$. The liquidity trap

[2]Note that there is also a role in this story for the coefficient of mean reversion in the expectations of backward-looking agents, d. This is discussed in detail in Appendix D.2.

now lasts less long for intermediate values of the fraction of backward-looking agents and deflationary spirals no longer occur unless both the fraction of backward-looking agents and the shock size are very large. This reflects that a shorter horizon implies faster recovery and shorter liquidity traps for any given combination of the shock size and the fraction of backward-looking agents.

Panels (c) and (d) of Figure 2 plot the cases of respectively $T = 16$ and $T = 32$. Here it can be seen that as the fraction of backward-looking agents is increased, the region of combinations of shock sizes and fractions of backward-looking agents that implies deflationary spirals becomes larger. This confirms the deflationary spiral observed in the green curve in Figure 3, that only arose for $T = 16$.

Panel (e) of Figure 2 plots the case of an infinite planning horizon. The results are very similar to those in panel (c) and (d). This is because backward-looking agents expect the economy to have more or less returned to steady state after 16 periods, so that backward-looking agents with a planning horizon longer than 16 periods do not base their consumption and pricing decisions on a longer sequence of low expected inflation and high expected real interest rates than agents with a planning horizon of 16 periods.

Note that in panel (e) the assumption is maintained that forward-looking agents do not anticipate revisions in expectations of backward-looking agents. In panel (f), I relax this assumption as a robustness check, and allow the forward-looking agents to become fully rational. Panel (e) and (f) look qualitatively similar, with short liquidity traps for small fractions of backward-looking agents and small shock-sizes, and deflationary spirals in case of larger fractions of backward-looking agents with a larger shock size. Forward-looking agents, however, now expect backward-looking agents to revise their expectations upward in case of a recovery. This leads to a slightly larger region of very short liquidity traps. The model equations used to create the graphs in panel (e) and (f) of Figure 2 are derived in Appendix E.

4 Fiscal stimulus in an expectations-driven liquidity trap

This section focuses on whether fiscal stimulus in the form of a temporary increase in government spending or cut in labor or consumption taxes can mitigate an expectations-driven liquidity trap. In particular, assume that the government implements a stimulus package in period 3 (the period after the start of the liquidity trap), and that the the stimulus package is persistent, with auto-correlation coefficient 0.7.[3]

4.1 Spending increases and tax cuts

I first assume in Section 4.1 that the size of the initial spending increase equal to the size of the initial negative shock to expectations. In Sections 4.2 and 4.3, I investigate what size of the stimulus package would be needed in order to prevent a deflationary spiral for respectively the benchmark calibration and the case of a positive inflation target. The sizes of the labor tax and consumption tax cuts will always be scaled by respectively $\frac{1}{\bar{w}}$ and $\frac{1}{1-\bar{g}}$, so that all stimulus measures have the same direct impact on the government's budget deficit, and hence are comparable.

The solid purple curves in Figure 4 reproduce the expectation driven liquidity trap of Figures 1. The other curves show the time series in case of a fiscal stimulus package starting from period 3. In particular, dashed green corresponds to a spending increase, dashed-dotted black corresponds to a labor tax cut, and dotted blue depicts the case of a cut in consumption taxes.[4]

First focusing on the dashed green case, the following can be observed. When gov-

[3] I find similar results if the stimulus is not persistent, or has a different auto-correlation coefficient. However, the lower the persistence in the stimulus package, the larger the initial stimulus needed to realize a required effect, which may lead to unrealistically large changes in fiscal variables.

[4] I have assumed that the implementation of the stimulus package is announced and anticipated in period 2. Assuming unanticipated fiscal stimulus in period 3 results in very similar graphs.

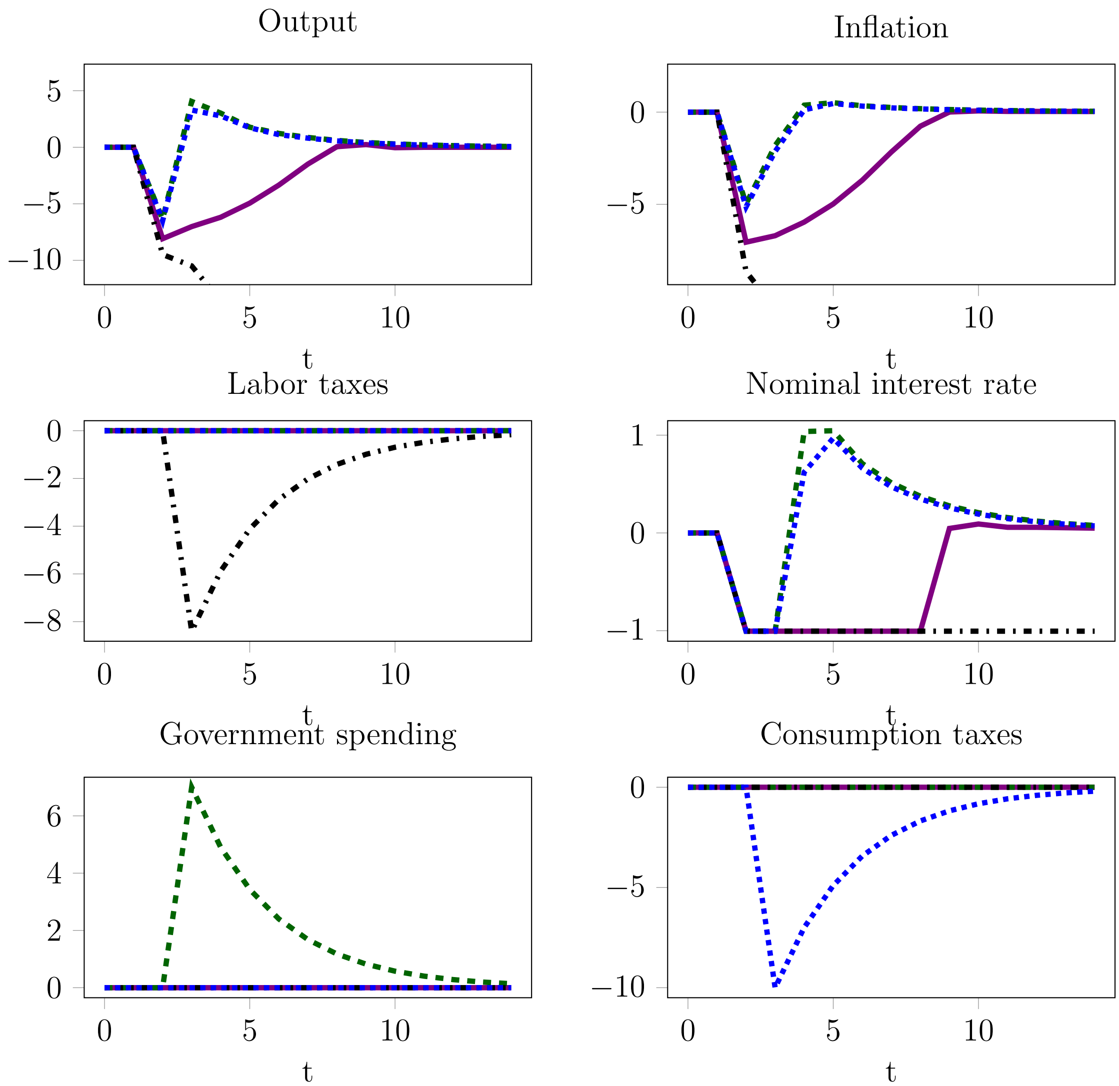


Figure 4: Expectations-driven liquidity trap for $T = 8$. The solid purple curves depict the case of no fiscal stimulus, while dashed green corresponds to an increase in government spending, dashed dotted black to a cut in labor taxes and dotted blue to a cut in consumption taxes.

ernment spending is increased in period 3, both output and inflation are increased considerably. Moreover, because forward-looking agents realize that the persistent spending increase will raise output and inflation also in the future, they raise their output and inflation expectations, and increase consumption and prices already somewhat in period 3. Backward-looking agents also raise their expectations, but only in period 4, because they then have observed higher output and inflation in period 3. This leads to an immediate end to the liquidity trap in period 4, making government spending increases a highly effective tool in an expectations-driven liquidity trap.

Next, consider the dashed-dotted black case of cutting labor taxes. As can be seen in Figure 4, this measure is deflationary. Under a binding zero lower bound, cutting labor taxes causes an increase in real interest rates, and, together with pessimistic expectations about the future, this implies that output falls on impact as well. All in all, this stimulus package makes the liquidity trap worse rather than better and in this particular example actually induces a deflationary spiral.

Finally, turn to the case of cutting consumption taxes, depicted in dotted blue. Unlike cutting labor taxes, this measure leads to inflationary pressures. This first of all directly mitigates the zero lower bound problem, and secondly implies a lower real interest rate and a further increase in consumption and output. The effectiveness of consumption tax cuts in raising output and inflation and ending the liquidity trap is very similar to that of increasing government spending.

The result that a spending increase or a consumption tax cut can quickly end a liquidity trap, while a labor tax cut is not effective is robust to the fraction of backward-looking agents and the shock size. This can be seen in panel (b) of Figures 5, 6 and 7, which reproduce Figure 2 for the cases of respectively spending increases, labor tax cuts, and consumption tax cuts. In these figures the size of the fiscal stimulus is increased to 3 times the size of the shock to expectations, to make it also an effective tool for larger fractions

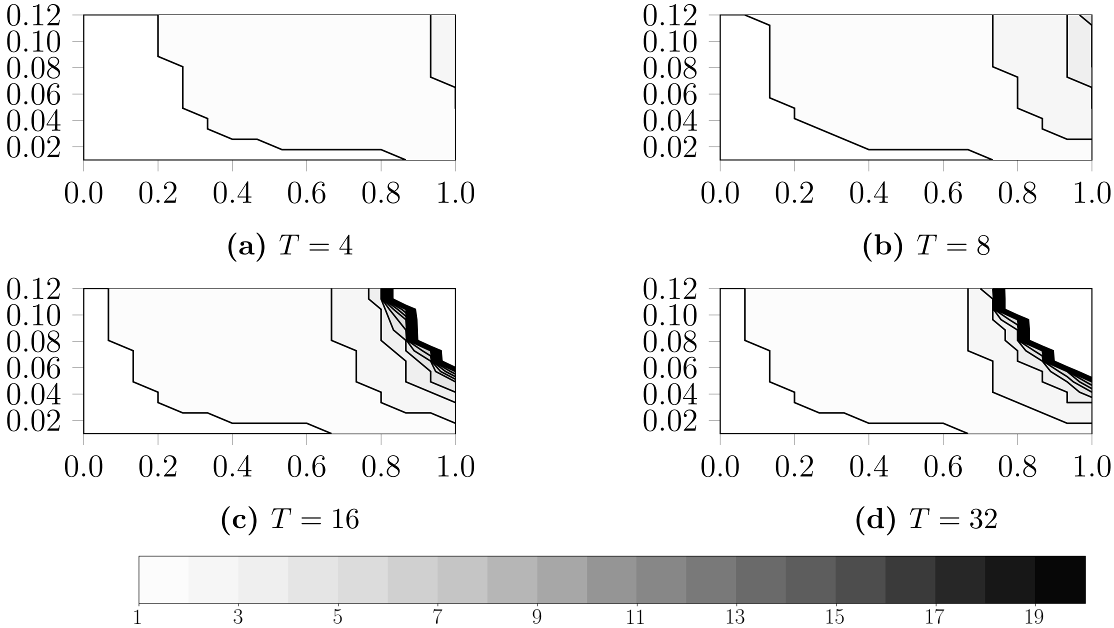


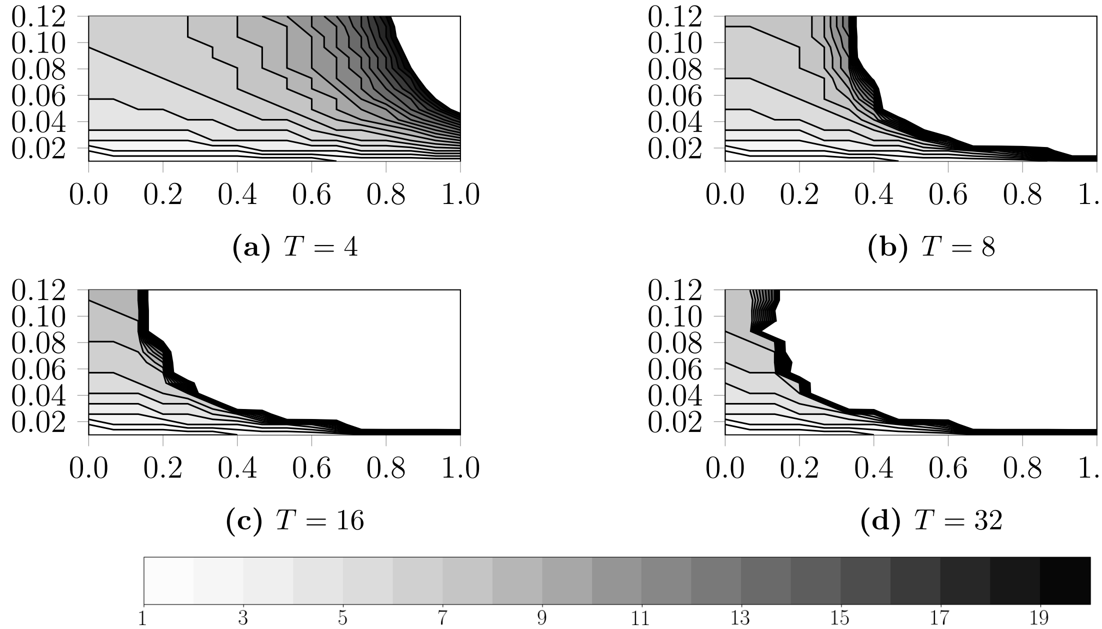
Figure 5: Length of liquidity trap in case of persistent **spending stimulus** for different fractions of backward-looking agents (x-axis), different sizes of the (non-persistent) negative shock to expectations (y-axis) and different horizons (the four panels).

of backward-looking agents.

Moreover, the same general result appears for different planning horizons. It can be seen in panel (a) of Figures 5, 6 and 7 that the liquidity trap is resolved more quickly under spending based stimulus and consumption tax cuts than under labor taxes cuts, also when the horizon is short. Comparing panels (d) and (e) of Figures 5, 6 and 7 with those of Figures 2, it can be seen that for longer horizons, government spending increases and consumption tax cuts shorten liquidity traps and reduce the risks of deflationary spirals, while labor tax cuts do not lead to improvements.

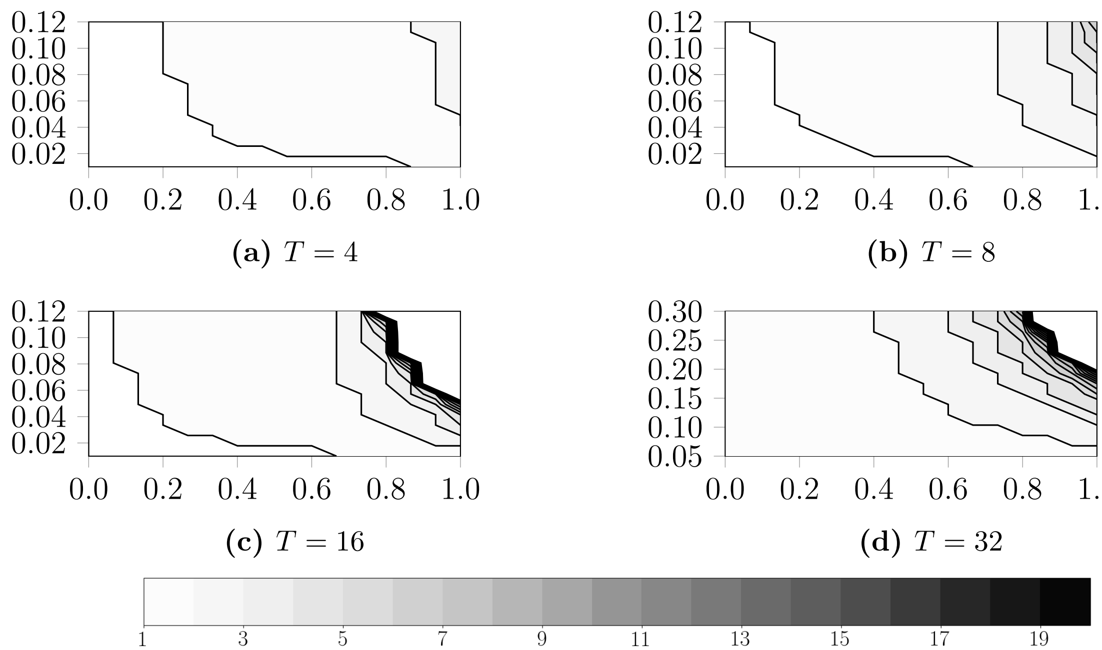
4.2 Size of the stimulus package

In Figure 4, the size of the spending stimulus was equal to the size of the negative shock to expectations, while in Figures 5 through 7 it was equal to three times that values. This is arguably somewhat ad hoc. Moreover, for both spending and tax based stimulus,



(a) $T = 4$ (b) $T = 8$ (c) $T = 16$ (d) $T = 32$

Figure 6: Length of liquidity trap in case of persistent **labor tax cuts** for different fractions of backward-looking agents (x-axis), different sizes of the (non-persistent) negative shock to expectations (y-axis) and different horizons (the four panels).



(a) $T = 4$ (b) $T = 8$ (c) $T = 16$ (d) $T = 32$

Figure 7: Length of liquidity trap in case of persistent **consumption tax cuts** for different fractions of backward-looking agents (x-axis), different sizes of the (non-persistent) negative shock to expectations (y-axis) and different horizons (the four panels).

deflationary spirals still occurred for longer horizons when the shock size and the fraction of backward-looking agents are large. Table 1 presents the size of the initial stimulus that is required to prevent a deflationary spiral for different shock sizes and different fractions of backward-looking agents, in case of a planning horizon of $T = 16$.[5] The table only presents the cases of a spending increase and of a consumption tax cut, because I find that a cut in labor taxes can never prevent a deflationary spiral, independent of its size.

Panel a: Increase in $\tilde{g}_t$ (government spending)

shock size\frac. BL	0.5	0.6	0.7	0.8	0.9	1
0.1	10	16	21	27	32	36
0.08	6	11	15	19	24	27
0.06	2	6	9	12	16	19
0.04	0	1	3	5	8	10
0.02	0	0	0	0	0	1

Panel b: Cut in $\tilde{\tau}_t^c(1-\bar{g})$ (consumption taxes)

shock size\frac. BL	0.5	0.6	0.7	0.8	0.9	1
0.3	11	17	23	29	34	39
0.25	7	12	16	21	26	30
0.2	2	6	10	13	17	20
0.15	0	1	3	6	8	11
0.1	0	0	0	0	0	1

Table 1: Required magnitude of fiscal stimulus (in percentage points) to prevent deflationary spiral for different shock sizes and fractions of backward-looking agents when agents have a planning horizon of $T = 16$.

It can be seen in panel a of Table 1 that for a shock size of 0.02, there is only a need for fiscal stimulus to prevent a liquidity trap for very large fractions of backward-looking agents. This is in line with panel (c) of Figure 2 where no deflationary spirals occur for small shock sizes and/or a small fraction of backward-looking agents. As the shock size and/or the fraction of backward-looking agents is increased in Table 1, larger spending increases are needed to prevent a deflationary spiral. For very large shocks and large fractions of backward-looking agents, the required size exceeds the size that was assumed

[5] As was found in Section 3, increasing the horizon above 16 (to 32 or even to infinity) does not drastically increase the occurrence of deflationary spirals. Therefore the required sizes of a stimulus package for longer horizons is very similar to the numbers presented in Table 1.

in Figure 5, which explains why deflationary spirals still occurred in panel (c) of that figure. This implies however, that with somewhat larger spending increases deflationary spirals could have been prevented all together in Figure 5.

Turning to panel b of Table 1, it can be seen that consumption taxes can also prevent deflationary spirals and that their required magnitudes (after re-scaling with steady the steady state consumption to output ratio, $\frac{\bar{C}}{\bar{Y}} = 1 - \bar{g}$, to make their direct impact on the governments budget deficit equal to that of a spending increase) are very similar, but slightly larger than the required government spending increases. Note however, that given the relatively low calibration of steady state consumption taxes, the cuts required to prevent deflationary spirals for large shocks and a large fraction of backward-looking agents may be unrealistically high and require consumption taxes to become negative. In this case, a combination of spending increases and consumption tax cuts may be a more realistic, better balanced way to proceed. Moreover, as shown in the next section, the required size of a stimulus package can be decreased with a positive inflation target.

4.3 Fiscal stimulus under positive inflation target

So far, I have assumed that the central bank has an inflation target of zero. It turns out that in the model, a higher inflation target decrease the occurrence, severity and duration of a liquidity trap for a given shock size. This is intuitive, as a positive inflation target implies a higher steady state nominal interest rate, so that there is more room to decrease interest rates until the zero lower bound is hit. Therefore, a liquidity trap of given severity only arises with larger shocks when the inflation target is set higher.

This also implies that for a given shock size and fraction of backward-looking agents, a smaller fiscal stimulus package is required to reduce a liquidity trap to a given duration, or to prevent a deflationary spiral. This is illustrated in Table 2 that reproduces Table 1 for the case of an annualized inflation target of 2%. It can be seen in Table 2 that fiscal

Panel a: Increase in $\tilde{g}_t$ (government spending)

shock size\frac. BL	0.5	0.6	0.7	0.8	0.9	1
0.1	7	12	17	22	27	31
0.08	2	7	11	15	19	22
0.06	0	1	5	8	11	13
0.04	0	0	0	1	3	5
0.02	0	0	0	0	0	0

Panel b: Cut in $\tilde{\tau}_t^c(1-\bar{g})$ (consumption taxes)

shock size\frac. BL	0.5	0.6	0.7	0.8	0.9	1
0.3	7	13	19	24	29	33
0.25	2	7	12	16	20	24
0.2	0	1	5	8	12	15
0.15	0	0	0	1	3	5
0.1	0	0	0	0	0	0

Table 2: Required magnitude of fiscal stimulus (in percentage points) to prevent deflationary spiral for different shock sizes and fractions of backward-looking agents when agents have a planning horizon of $T = 16$ and the central bank targets inflation of annualized 2%.

intervention ro prevent deflationary spirals is only required when both the shock size and the fraction of backward-looking agents are quite large. Moreover, the required size of the stimulus package is considerably reduced compared to Table 1, where the inflation target was zero. This holds for both spending increases and consumption tax cuts. As before, labor tax cuts are not effective in preventing liquidity traps, and this case is therefore not shown in Table 2.

5 Conclusion

I present a New Keynesian model with two forms of bounded rationality. First of all, all agents in the economy have a finite planning horizon and are not able to base their consumption and pricing decisions upon considerations and expectations about the infinite future. Secondly, while one fraction of agents is forward-looking and uses the model equations to form expectations, another fraction of agents forms expectations in a backward-looking manner, based on the most recently observed state of the economy. They expect a similar economic situation to continue in the short run, but expect mean reversion to the target steady state in the medium to long run.

The presence of backward-looking agents in the economy can result in a liquidity trap of multiple periods, driven by expectations, after a single negative shock to inflation and output expectations of backward-looking agents. The duration of such an liquidity trap crucially depends on the fraction of backward-looking agents in the economy, the size of the shock that triggered the liquidity trap, and agents' planning horizons. When these quantities are low, the liquidity trap lasts at most one or two periods. Expectations-driven liquidity traps of longer duration can arise if the planing horizon is still relatively short, but the fraction of backward-looking agents becomes larger. When both the planning horizon and the fraction of backward-looking agents are large, it can occur that the economy never recovers from the liquidity trap, but instead falls in a deflationary spiral.

I show that fiscal stimulus in the form of a spending increase or a cut in consumption taxes is very effective in reducing the length of the liquidity trap. Moreover, when the stimulus is of the appropriate size, deflationary spirals can always be prevented. Labor tax cuts on the other hand, are not effective in reducing the length of the liquidity traps, and are not able to prevent deflationary spirals, independent of the size of the tax cut. The intuition for this is that, as in a standard liquidity trap driven by fundamentals, spending increases and consumption tax cuts are inflationary in my expectations-driven liquidity traps, while labor tax cuts are deflationary. This result is in contrast with the findings of Mertens and Ravn (2014) who find that in a liquidity trap driven by a sunspot shock spending increases are deflationary while labor tax cuts are inflationary.

Finally, I find that a higher inflation target reduces the severity and duration of liquidity traps. Therefore, smaller fiscal stimulus packages are needed to mitigate liquidity traps and prevent deflationary spiral when the monetary authority targets higher inflation.

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A Steady state

In this section the steady state of the non-linear model is derived, where the preference shock is assumed to be constant at $\xi = 1$.

From the consumer Euler equation it follows that in this steady state we must have

$$\frac{1+\bar{i}}{\bar{\Pi}} = \frac{1}{\beta} \tag{32}$$

Furthermore, from (63) it follows that

$$\bar{H} = \bar{s}\bar{Y} \tag{33}$$

Next, we can solve the steady state aggregate resource constraint, (21), for consumption, and write

$$\bar{C} = \bar{Y}(1-\bar{g}) \tag{34}$$

Plugging in these steady state labor and consumption levels in the steady state version of the optimal labor/consumption trade off gives

$$\bar{mc} = \bar{w} = \frac{\bar{s}^{\eta}\bar{Y}^{\eta+\sigma}(1-\bar{g})^{\sigma}(1+\bar{\tau}^{c})}{1-\bar{\tau}^{l}} \tag{35}$$

So that steady state output can be written as

$$\bar{Y} = \left(\frac{\bar{mc}(1-\bar{\tau}^{l})}{\bar{s}^{\eta}(1-\bar{g})^{\sigma}(1+\bar{\tau}^{c})}\right)^{\frac{1}{\eta+\sigma}} \tag{36}$$

For the relative optimal price, we write (58) as

$$\bar{d} = \left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} \tag{37}$$

For price dispersion, we then write

$$\bar{s} = \frac{(1-\omega)}{1-\omega\bar{\Pi}^{\theta}}\bar{d}^{-\theta} = \frac{(1-\omega)}{1-\omega\bar{\Pi}^{\theta}}\left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{\theta}{\theta-1}} \tag{38}$$

Evaluating (50) at the steady state gives

$$\bar{mc} = \bar{d}\frac{\theta-1}{\theta}\frac{\sum_{s=0}^{T}\omega^s\beta^s\bar{\Pi}^{s(\theta-1)}}{\sum_{s=0}^{T}\omega^s\beta^s\bar{\Pi}^{s(\theta)}} = \left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}}\frac{\theta-1}{\theta}\frac{(1-\omega\beta\Pi^{\theta})(1-(\omega\beta\Pi^{\theta-1})^T)}{(1-(\omega\beta\Pi^{\theta})^T)(1-\omega\beta\Pi^{\theta-1})} \tag{39}$$

Firm profits we can write as

$$\bar{\Xi} = (1-\bar{w}\bar{s})\bar{Y} \tag{40}$$

Then we turn to the government budget constraint. In steady state (20) reduces to

$$\frac{\beta\bar{b}}{\bar{\Pi}} = (1+\bar{\tau}^c)\bar{g} - \bar{\tau}^l\bar{w}\bar{s} - \bar{\tau}^c - \frac{\bar{LS}}{\bar{Y}} + \frac{\bar{b}}{\bar{\Pi}}, \tag{41}$$

which gives

$$\bar{b} = \bar{\Pi}\frac{(\bar{\tau}^l\bar{w}\bar{s} + \bar{\tau}^c + \frac{\bar{LS}}{\bar{Y}} - (1+\bar{\tau}^c)\bar{g})}{1-\beta}, \tag{42}$$

B Log-linearized model

In this Section, I log-linearize the model equations around the steady state.

B.1 Households

The log linearized optimality conditions of the households (including budget constraints) are given by

$$\hat{C}^i_{\tau} = \hat{C}^i_{\tau+1} - \frac{1}{\sigma}(i_{\tau} - \hat{\pi}_{\tau+1} + \xi_{\tau+1} - \xi_{\tau} - \frac{\tilde{\tau}^c_{\tau+1} - \tilde{\tau}^c_{\tau}}{1+\bar{\tau}^c}), \qquad \tau = t, t+1, ...t+T-1 \tag{43}$$

$$\tilde{b}^i_{t+T+1} = \frac{u_b}{1-\beta}\hat{C}^i_{t+T} + \frac{u_b}{(1-\beta)\sigma}\frac{\tilde{\tau}^c_{t+T}}{1+\bar{\tau}^c} + \frac{u_b}{(1-\beta)\sigma}E^i_t i_{t+T} - \frac{u_b}{(1-\beta)\sigma}\xi_{t+T}, \tag{44}$$

$$\eta\hat{H}^i_\tau = -\sigma\hat{C}^i_\tau - \frac{\tilde{\tau}^c_\tau}{1+\bar{\tau^c}} - \frac{\tilde{\tau}^l_\tau}{1-\bar{\tau}^l} + \hat{w}_\tau, \qquad \tau = t, t+1, ...t+T \tag{45}$$

$$\begin{aligned}\tilde{b}^i_{\tau+1} =& \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}((1-\bar{\tau}^l)(E^i_t\hat{w}_\tau + \hat{H}^i_\tau) - E^i_t\tilde{\tau}^l_\tau) + \frac{1}{\beta}\tilde{b}^i_\tau + \bar{b}(\hat{i}_\tau - \frac{1}{\beta}E^i_t\hat{\pi}_\tau) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}\beta}E^i_t\hat{\Xi}_\tau \\ &- \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\beta}E^i_t\hat{LS}_\tau - \frac{(1-\bar{g})\bar{\Pi}}{\beta}\left((1+\bar{\tau}^c)\hat{C}^i_\tau + \tilde{\tau}^c_\tau\right), \qquad \tau = t, t+1, ...t+T\end{aligned} \tag{46}$$

with

$$u_b = (1-\bar{g})(1+\bar{\tau}^c)\bar{\Pi} \tag{47}$$

where it is used that $\bar{H} = \bar{s}\bar{Y}$, $\frac{\bar{C}}{\bar{Y}} = 1-\bar{g}$ and $\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{1+\bar{\tau}^c}\frac{\bar{Y}\bar{b}}{\Pi} = \bar{C}$

Iterating the budget constraint T periods, and using the first order conditions of the household, the following equation can be derived, that describes a households optimal consumption decision in period t.

$$
\begin{aligned}
&\left(\frac{\beta^{T+1}}{1-\beta}u_b + \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t^i = \\
&\tilde{b}_t^i + \bar{w}\bar{s}\bar{\Pi}(1-\bar{\tau}^l)\sum_{s=0}^{T}\beta^s\left((1+\frac{1}{\eta})(E_t^i\hat{w}_{t+s} - \frac{E_t^i\tilde{\tau}_{t+s}^l}{1-\bar{\tau}^l}) - \frac{1}{\eta}\frac{E_t^i\tilde{\tau}_{t+s}^c}{1+\bar{\tau}^c}\right) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{\Xi}_{t+s}) \\
&-\frac{\bar{LS}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{LS}_{t+s}) - \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t^i i_{t+j} - E_t^i\hat{\pi}_{t+j+1}) \\
&-\left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\left(\frac{1}{\sigma}(E_t^i\xi_{t+s} - E_t^i\xi_t - \frac{E_t^i\tilde{\tau}_{t+s}^c - E_t^i\tilde{\tau}_t^c}{1+\bar{\tau}^c})\right) \\
&-(1-\bar{g})\bar{\Pi}\sum_{s=0}^{T}\beta^s\left(E_t^i\tilde{\tau}_{t+s}^c\right) + \bar{b}\sum_{s=0}^{T}\beta^s(\beta E_t^i\hat{i}_{t+s} - E_t^i\hat{\pi}_{t+s}) \qquad (48) \\
&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\sum_{j=0}^{T-1}(E_t^i i_{t+j} - E_t^i\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}E_t^i i_{t+T} + \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\xi_t - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\frac{\tilde{\tau}_t^c}{1+\bar{\tau}^c}
\end{aligned}
$$

Aggregating this equation over all households yields an expression for aggregate consumption as a function of aggregate expectations about aggregate variables, only.

$$
\begin{aligned}
&\left(\frac{\beta^{T+1}}{1-\beta}u_b + \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t = \\
&\tilde{b}_t + \bar{w}\bar{s}\bar{\Pi}(1-\bar{\tau}^l)\sum_{s=0}^{T}\beta^s\left((1+\frac{1}{\eta})(\bar{E}_t\hat{w}_{t+s} - \frac{\bar{E}_t\tilde{\tau}_{t+s}^l}{1-\bar{\tau}^l}) - \frac{1}{\eta}\frac{\bar{E}_t\tilde{\tau}_{t+s}^c}{1+\bar{\tau}^c}\right) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\Xi}_{t+s}) \\
&-\frac{\bar{LS}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) - \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\sum_{j=0}^{s-1}\frac{1}{\sigma}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) \\
&-\left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\left(\frac{1}{\sigma}(\bar{E}_t\xi_{t+s} - \bar{E}_t\xi_t - \frac{\bar{E}_t\tilde{\tau}_{t+s}^c - \bar{E}_t\tilde{\tau}_t^c}{1+\bar{\tau}^c})\right) \\
&-(1-\bar{g})\bar{\Pi}\sum_{s=0}^{T}\beta^s\left(\bar{E}_t\tilde{\tau}_{t+s}^c\right) + \bar{b}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s} - \bar{E}_t\hat{\pi}_{t+s}) \qquad (49) \\
&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\bar{E}_t i_{t+T} + \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\xi_t - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\frac{\tilde{\tau}_t^c}{1+\bar{\tau}^c}
\end{aligned}
$$

B.2 Firms

Equation (17) can be written as

$$\frac{p_t^*(j)}{P_t}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\theta-1} Y_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta-1}}\bar{Y}\bar{\lambda}\left(\frac{\bar{\Pi}P_{t+T}}{P_t}\right)^{\theta-1}\right] \tag{50}$$

$$=\frac{\theta}{\theta-1}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\theta} Y_{t+s} mc_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta}}\bar{Y}\bar{\lambda}\bar{mc}\left(\frac{\bar{\Pi}P_{t+T}}{P_t}\right)^{\theta}\right]$$

Eliminating prices, we can instead write the equation in terms of $d_t(j) = \frac{p_t^*(j)}{P_t}$ and in terms of inflation as

$$d_t(j)\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\prod_{j=1}^{s}\Pi_{t+j}\right)^{\theta-1} Y_{t+s} + \frac{(\omega\beta)^{T+1}\bar{\Pi}^{\theta-1}}{1-\omega\beta\bar{\Pi}^{\theta-1}}\bar{Y}\bar{\lambda}\left(\prod_{j=1}^{T}\Pi_{t+j}\right)^{\theta-1}\right]$$

$$=\frac{\theta}{\theta-1}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\prod_{j=1}^{s}\Pi_{t+j}\right)^{\theta} Y_{t+s} mc_{t+s} + \frac{(\omega\beta)^{T+1}\bar{\Pi}^{\theta}}{1-\omega\beta\bar{\Pi}^{\theta}}\bar{Y}\bar{\lambda}\bar{mc}\left(\prod_{j=1}^{T}\Pi_{t+j}\right)^{\theta}\right], \tag{51}$$

Log linearizing (51) gives

$$\hat{d}_t(j) = \tilde{E}_t^j \sum_{s=0}^{T}\left(\frac{(c_1)^s}{s_1} - \frac{(c_2)^s}{s_2}\right)\left(\hat{Y}_{t+s} - \sigma\hat{C}_{t+s} - \frac{\tilde{\tau}_{t+s}^c}{1+\bar{\tau}^c} + \hat{\xi}_{t+s}\right) + \tilde{E}_t^j \sum_{s=0}^{T}\frac{(c_1)^s}{s_1}\hat{mc}_{t+s} \tag{52}$$

$$+ \tilde{E}_t^j \sum_{s=1}^{T}\left(\theta(c_1)^s - (\theta-1)(c_2)^s\right)\hat{\pi}_{t+s}$$

with

$$c_1 = \omega\beta\bar{\Pi}^{\theta} \tag{53}$$

$$c_2 = \omega\beta\bar{\Pi}^{\theta-1} \tag{54}$$

$$s_1 = \frac{1}{1 - \omega\beta\bar{\Pi}^{\theta}} \tag{55}$$

$$s_2 = \frac{1}{1 - \omega\beta\bar{\Pi}^{\theta-1}} \tag{56}$$

Aggregating (52) yields

$$\begin{aligned}\int_0^1 \hat{d}_t(j)\mathrm{d}j =& \bar{E}_t \sum_{s=0}^{T} \left(\frac{(c_1)^s}{s_1} - \frac{(c_2)^s}{s_2}\right) \left(\hat{Y}_{t+s} - \sigma\hat{C}_{t+s} - \frac{\tilde{\tau}^c_{t+s}}{1+\bar{\tau^c}} + \hat{\xi}_{t+s}\right) + \bar{E}_t \sum_{s=0}^{T} \frac{(c_1)^s}{s_1} \hat{mc}_{t+s} \\ & + \bar{E}_t \sum_{s=1}^{T} \left(\theta(c_1)^s - (\theta-1)(c_2)^s\right) \hat{\pi}_{t+s}\end{aligned} \tag{57}$$

Next, dividing by P_t, (18) can be written as

$$1 = \omega\Pi_t^{\theta-1} + (1-\omega)\int_0^1 d_t(j)^{1-\theta}\mathrm{d}j, \tag{58}$$

Log linearizing, this implies

$$\hat{\pi}_t = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}} \int_0^1 \hat{d}_t(j)\mathrm{d}j. \tag{59}$$

Plugging in in (57) gives

$$\begin{aligned}\hat{\pi}_t =& \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}} \left[\bar{E}_t \sum_{s=0}^{T} \left(\frac{(c_1)^s}{s_1} - \frac{(c_2)^s}{s_2}\right) \left(\hat{Y}_{t+s} - \sigma\hat{C}_{t+s} - \frac{\tilde{\tau}^c_{t+s}}{1+\bar{\tau^c}} + \hat{\xi}_{t+s}\right) + \bar{E}_t \sum_{s=0}^{T} \frac{(c_1)^s}{s_1} \hat{mc}_{t+s}\right. \\ & \left. + \bar{E}_t \sum_{s=1}^{T} \left(\theta(c_1)^s - (\theta-1)(c_2)^s\right) \hat{\pi}_{t+s}\right]\end{aligned} \tag{60}$$

B.3 Final equations

To complete the model, I first log-linearize the government budget constraint, (20), to

$$\tilde{b}_{t+1} = \frac{\bar{\Pi}}{\beta}\tilde{g}_t - \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}(\bar{\tau}^l(\hat{w}_t + \hat{H}_t) + \tilde{\tau}^l_t) - \frac{\bar{\Pi}}{\beta}(1-\bar{g})(\bar{\tau}^c\hat{C}_t + \tilde{\tau}^c_t) - \frac{\bar{\Pi}\bar{LS}}{\beta\bar{Y}}\hat{LS}_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t), \quad (61)$$

Next, we can log linearize the market clearing condition, (21)

$$\hat{Y}_t = (1-\bar{g})\hat{C}_t + \tilde{g}_t, \quad (62)$$

Next, I turn to aggregate labor

$$H_t = \int_0^1 H_t(j)\mathrm{d}j = \int_0^1 Y_t(j)\mathrm{d}j = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} \mathrm{d}jY_t = s_tY_t, \quad (63)$$

where $s_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} \mathrm{d}j$ is price dispersion in the economy in period t. Linearizing this equation and aggregating (45) wages and marginal costs can be written as

$$\begin{aligned} \hat{mc}_t = \hat{w}_t &= \eta\hat{H}_t + \sigma\hat{C}_t + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} \quad (64) \\ &= (\eta + \frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} + \eta\hat{s}_t \end{aligned}$$

Because all prices in the economy where set at different dates by the Calvo mechanism, price dispersion can be written as

$$\begin{aligned} s_t &= (1-\omega)\int_0^1 \left(\frac{p^*_t(j)}{P_t}\right)^{-\theta}\mathrm{d}j + \omega(1-\omega)\int_0^1 \left(\frac{p^*_{t-1}(j)}{P_t}\right)^{-\theta}\mathrm{d}j + \omega^2(1-\omega)\int_0^1 \left(\frac{p^*_{t-2}(j)}{P_t}\right)^{-\theta}\mathrm{d}j + ... \\ &= (1-\omega)\sum_{i=0}^{\infty}\omega^i\int_0^1 \left(\frac{p^*_{t-i}(j)}{P_t}\right)^{-\theta}\mathrm{d}j \quad (65) \end{aligned}$$

We can therefore write price dispersion as

$$\begin{aligned} s_t &= (1-\omega)\int_0^1 \left(\frac{p_t^*(j)}{P_t}\right)^{-\theta} \mathrm{d}j + \omega\Pi_t^\theta(1-\omega)\sum_{i=0}^{\infty}\omega^i\int_0^1\left(\frac{p_{t-1-i}^*(j)}{P_{t-1}}\right)^{-\theta}\mathrm{d}j \\ &= (1-\omega)\int_0^1 d_t(j)^{-\theta}\mathrm{d}j + \omega\Pi_t^\theta s_{t-1} \end{aligned} \tag{66}$$

Price dispersion is log linearized to

$$\hat{s}_t = -\theta(1-\omega\bar{\Pi}^\theta)\int_0^1 \hat{d}_t(j)\mathrm{d}j + \theta\omega\bar{\Pi}^\theta\hat{\pi}_t + \omega\bar{\Pi}^\theta\hat{s}_{t-1} \tag{67}$$

Using (59), this can be written as

$$\hat{s}_t = \frac{\theta\omega\bar{\Pi}^{\theta-1}}{1-\omega\bar{\Pi}^{\theta-1}}(\bar{\Pi}-1)\hat{\pi}_t + \omega\bar{\Pi}^\theta\hat{s}_{t-1} \tag{68}$$

Finally, we can write real aggregate firm profits as

$$\Xi_t = \int_0^1 \Xi_t(j)\mathrm{d}j = \int_0^1 Y_t(j)\frac{P_t(j)}{P_t} - mc_t Y_t(j)\mathrm{d}j = (1-mc_t s_t)Y_t, \tag{69}$$

which can be log-linearized to

$$\hat{\Xi}_t = \hat{Y}_t - \frac{\bar{s}\bar{w}}{1-\bar{s}\bar{w}}\left(\hat{mc}_t + \hat{s}_t\right). \tag{70}$$

Using (21) in (49) results in an expression for aggregate output.

$$
\begin{aligned}
\hat{Y}_t = {} & \frac{1}{\rho}\tilde{b}_t + g_t + \delta \sum_{s=0}^{T} \beta^s ((1-\bar{\tau}^l)\bar{E}_t \hat{w}_{t+s} - \bar{E}_t \tilde{\tau}^l_{t+s}) + \frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho} \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{\Xi}_{t+s}) \\
& - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{LS}_{t+s}) - \mu \sum_{s=1}^{T} \beta^s \sum_{j=0}^{s-1} (\bar{E}_t i_{t+j} - \bar{E}_t \hat{\pi}_{t+j+1}) + \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\beta \bar{E}_t \hat{i}_{t+s} - \bar{E}_t \hat{\pi}_{t+s}) \\
& - \frac{\beta^{T+1}}{1-\beta} \frac{u_b}{\sigma\rho} \sum_{j=0}^{T-1} (\bar{E}_t i_{t+j} - \bar{E}_t \hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta} \frac{u_b}{\sigma\rho} \bar{E}_t i_{t+T} \\
& \mu_\xi \xi_t - \mu \sum_{s=1}^{T} \beta^s \bar{E}_t \xi_{t+s} - \mu_c \tilde{\tau}^c_t - \frac{\bar{\Pi}(1-\bar{g})}{\rho} \left(1 - \frac{1}{\sigma}\right) \sum_{s=1}^{T} \beta^s \bar{E}_t \tilde{\tau}^c_{t+s}
\end{aligned} \tag{71}
$$

$$\delta = \frac{\bar{w}\bar{s}\bar{\Pi}}{\rho} \frac{\eta+1}{\eta} \tag{72}$$

$$\mu = \frac{\bar{\Pi}}{\rho} \left(\frac{\bar{w}\bar{s}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{1-\bar{g}}{\sigma} \right) \tag{73}$$

$$\mu_\xi = \frac{\bar{\Pi}}{\rho} \left(\frac{\beta - \beta^{T+1}}{1-\beta} \frac{\bar{w}\bar{s}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{(1-\bar{g})\beta}{\sigma(1-\beta)} \right) \tag{74}$$

$$\mu_c = \frac{\bar{\Pi}}{\rho} \left(\frac{\beta - \beta^{T+1}}{1-\beta} \frac{\bar{w}\bar{s}}{\eta} \frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} + \frac{(1-\bar{g})\beta}{\sigma(1-\beta)} + (1-\bar{g}) \right) \tag{75}$$

$$\rho = \frac{1}{1-\bar{g}} \left[\frac{\beta^{T+1}}{1-\beta} u_b + \left(\frac{\sigma}{\eta} \bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g}) \right) \bar{\Pi} \frac{1-\beta^{T+1}}{1-\beta} \right] \tag{76}$$

I now assume that agents know, or have learned about the above relations between aggregate variables (which hold in every period). Therefore, expectations about wages and profits can be substituted for, using (64) and (70). This gives the following system of 3 equations that, together with a specification of monetary and fiscal policy and price dispersion, completely describe our model

$$
\begin{aligned}
(1-\nu_y)\hat{Y}_t = &\frac{1}{\rho}\tilde{b}_t + g_t + \nu_\tau \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{\tau}^l_{t+s}) + \nu_g \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{g}_{t+s}) + \nu_y \sum_{s=1}^{T} \beta^s (\bar{E}_t \hat{Y}_{t+s}) \\
&+ \nu_s \sum_{j=0}^{T} \beta^j (\bar{E}_t \hat{s}_{t+j}) - \mu \sum_{s=1}^{T} \beta^s \sum_{j=1}^{s} (\bar{E}_t i_{t+j-1} - \bar{E}_t \hat{\pi}_{t+j}) + \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\beta \bar{E}_t \hat{i}_{t+s} - \bar{E}_t \hat{\pi}_{t+s}) \\
&- \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho} \sum_{j=0}^{T-1} (\bar{E}_t i_{t+j} - \bar{E}_t \hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho} \bar{E}_t i_{t+T} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s (\bar{E}_t \hat{LS}_{t+s}) \\
&\mu_\xi \xi_t - \mu \sum_{s=1}^{T} \beta^s \bar{E}_t \xi_{t+s} + \nu_{c1} \tilde{\tau}^c_t + \nu_{c2} \sum_{s=1}^{T} \beta^s \bar{E}_t \tilde{\tau}^c_{t+s}
\end{aligned} \tag{77}
$$

$$
\begin{aligned}
\hat{\pi}_t = &\bar{E}_t \sum_{s=0}^{T} \left(\kappa_{y1}(c_1)^s + \kappa_{y2}(c_2)^s\right) \hat{Y}_{t+s} + \kappa_g \bar{E}_t \sum_{s=0}^{T} (c_2)^s \tilde{g}_{t+s} + \kappa_s \bar{E}_t \sum_{s=0}^{T} (c_1)^s \hat{s}_{t+s} \\
&+ \kappa_c \bar{E}_t \sum_{s=0}^{T} (c_2)^s \tilde{\tau}^c_{t+s} + \kappa_\tau \bar{E}_t \sum_{s=0}^{T} (c_1)^s \tilde{\tau}^l_{t+s} + \bar{E}_t \sum_{s=1}^{T} \left(\kappa_{\pi1}(c_1)^s + \kappa_{\pi2}(c_2)^s\right) \hat{\pi}_{t+s} \\
&+ \bar{E}_t \sum_{s=0}^{T} \left(\kappa_{\xi1}(c_1)^s + \kappa_{\xi2}(c_2)^s\right) \hat{\xi}_{t+s}
\end{aligned} \tag{78}
$$

$$
\begin{aligned}
\tilde{b}_{t+1} = &\frac{\bar{\Pi}}{\beta}\tilde{g}_t - \frac{\bar{\Pi}}{\beta}\bar{\tau}^c(\hat{Y}_t - \tilde{g}_t) - \frac{\bar{\Pi}}{\beta}(1-\bar{g})\tilde{\tau}^c_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t) - \frac{\bar{\Pi}\bar{LS}}{\beta\bar{Y}}\hat{LS}_t \\
&- \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}\left[\bar{\tau}^l\left((1+\eta+\frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} + (1+\eta)\hat{s}_t\right) + \tilde{\tau}^l_t\right],
\end{aligned} \tag{79}
$$

with

$$
\nu_y = \frac{(1-\bar{s}\bar{w})\bar{\Pi}}{\rho} + \left(\delta(1-\bar{\tau}^l) - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho}\right)(\eta + \frac{\sigma}{1-\bar{g}}), \tag{80}
$$

$$
\nu_g = \left(\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho} - \delta(1-\bar{\tau}^l)\right)\frac{\sigma}{1-\bar{g}}, \tag{81}
$$

$$\nu_\tau = -\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1-\bar{\tau}^l)}, \tag{82}$$

$$\nu_s = -\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho}(\eta+1)\bar{\tau}^l, \tag{83}$$

$$\nu_{c1} = \delta\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1+\bar{\tau}^c)} - \mu_c, \tag{84}$$

$$\nu_{c2} = \delta\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1+\bar{\tau}^c)} - \frac{\bar{\Pi}(1-\bar{g})}{\rho}\left(1-\frac{1}{\sigma}\right), \tag{85}$$

$$\kappa_{y1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1+\eta)(1-\omega\beta\bar{\Pi}^{\theta}) \tag{86}$$

$$\kappa_{y2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\frac{\sigma}{1-\bar{g}})(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{87}$$

$$\kappa_g = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{\sigma}{(1-\bar{g})}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{88}$$

$$\kappa_s = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\eta(1-\omega\beta\bar{\Pi}^{\theta}) \tag{89}$$

$$\kappa_c = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{1}{(1+\bar{\tau}^c)}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{90}$$

$$\kappa_\tau = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{1}{(1-\bar{\tau})}(1-\omega\beta\bar{\Pi}^{\theta}) \tag{91}$$

$$\kappa_{\pi1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\theta \tag{92}$$

$$\kappa_{\pi2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(\theta-1) \tag{93}$$

$$\kappa_{\xi1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\omega\beta\bar{\Pi}^{\theta}) \tag{94}$$

$$\kappa_{\xi2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{95}$$

B.4 Zero inflation target

In case of $\bar{\Pi} = 0$ the above model reduces to

$$
\begin{aligned}
&(1-\nu_{y0})\hat{Y}_t = \frac{1}{\rho_0}\tilde{b}_t + g_t + \nu_{\tau 0}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\tau}^l_{t+s}) + \nu_{g0}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{g}_{t+s}) + \nu_{y0}\sum_{s=1}^{T}\beta^s(\bar{E}_t\hat{Y}_{t+s}) \\
&-\mu_0\sum_{s=1}^{T}\beta^s\sum_{j=1}^{s}(\bar{E}_t i_{t+j-1} - \bar{E}_t\hat{\pi}_{t+j}) + \frac{\bar{b}}{\rho_0}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s} - \bar{E}_t\hat{\pi}_{t+s}) \\
&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho_0}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho_0}\bar{E}_t i_{t+T} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) \qquad (96) \\
&\mu_{\xi 0}\xi_t - \mu_0\sum_{s=1}^{T}\beta^s\bar{E}_t\xi_{t+s} + \nu_{c10}\tilde{\tau}^c_t + \nu_{c20}\sum_{s=1}^{T}\beta^s\bar{E}_t\tilde{\tau}^c_{t+s}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_t =&\kappa(\eta + \frac{\sigma}{1-\bar{g}})\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\hat{Y}_{t+s} - \frac{\kappa\sigma}{1-\bar{g}}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{g}_{t+s} \qquad (97) \\
&+\frac{\kappa}{1+\bar{\tau}^c}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{\tau}^c_{t+s} + \frac{\kappa}{1-\bar{\tau}^l}\sum_{s=0}^{T}\omega^s\beta^s\bar{E}_t\tilde{\tau}^l_{t+s} + \frac{1-\omega}{\omega}\sum_{s=1}^{T}\omega^s\beta^s\bar{E}_t\hat{\pi}_{t+s},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{b}_{t+1} =&\frac{1}{\beta}\tilde{g}_t - \frac{\bar{\tau}^c}{\beta}(\hat{Y}_t - \tilde{g}_t) - \frac{1-\bar{g}}{\beta}\tilde{\tau}^c_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t) - \frac{\bar{LS}}{\beta\bar{Y}}\hat{LS}_t \qquad (98) \\
&-\frac{\bar{w}}{\beta}\left[\bar{\tau}^l\left((1+\eta+\frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l}\right) + \tilde{\tau}^l_t\right],
\end{aligned}
$$

$$\delta_0 = \frac{\bar{w}}{\rho_0}\frac{\eta+1}{\eta}, \qquad (99)$$

$$\mu_0 = \frac{1}{\rho_0}\left(\frac{\bar{w}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{1-\bar{g}}{\sigma}\right), \qquad (100)$$

$$\delta_{c0} = \frac{1-\beta^{T+1}}{1-\beta}\frac{\bar{w}}{\rho_0\eta}\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} \qquad (101)$$

$$\mu_{\xi 0} = \frac{1}{\rho_0}\left(\frac{\beta-\beta^{T+1}}{1-\beta}\frac{\bar{w}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{(1-\bar{g})\beta}{\sigma(1-\beta)}\right) \tag{102}$$

$$\nu_{c10} = \delta_0\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{w}}{\rho_0(1+\bar{\tau}^c)} - \mu_{c0}, \tag{103}$$

$$\nu_{c20} = \delta_0\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{w}}{\rho_0(1+\bar{\tau}^c)} - \frac{(1-\bar{g})}{\rho_0}\left(1-\frac{1}{\sigma}\right), \tag{104}$$

$$\mu_{c0} = \frac{1}{\rho_0}\left(\frac{\beta-\beta^{T+1}}{1-\beta}\frac{\bar{w}}{\eta}\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} + \frac{(1-\bar{g})\beta}{\sigma(1-\beta)} + (1-\bar{g})\right) \tag{105}$$

$$\rho_0 = \frac{1}{1-\bar{g}}\left[\frac{\beta^{T+1}}{1-\beta}u_b + \left(\frac{\sigma}{\eta}\bar{w}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\frac{1-\beta^{T+1}}{1-\beta}\right]. \tag{106}$$

$$\nu_{y0} = \frac{1-\bar{w}}{\rho_0} + \left(\delta(1-\bar{\tau}^l) - \frac{\bar{w}}{\rho_0}\right)(\eta + \frac{\sigma}{1-\bar{g}}), \tag{107}$$

$$\nu_{g0} = \left(\frac{\bar{w}}{\rho_0} - \delta_0(1-\bar{\tau}^l)\right)\frac{\sigma}{1-\bar{g}}, \tag{108}$$

$$\nu_{\tau 0} = -\frac{\bar{w}}{\rho_0(1-\bar{\tau}^l)}, \tag{109}$$

$$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}. \tag{110}$$

C Expectation algorithm of forward-looking agents

Forward-looking agents use a solution algorithm where they start with the final period of their horizon and then solve the model backward. In each step they will compute a solution of the endogenous variables of that particular period in terms of the state variables of that period.

Starting with the final period of their horizon, forward-looking agents take the model equations of period $t+T$. However, in the IS and Phillips curves of that period (Equations (24) and (25) forwarded T periods), finite sums with expectations about period $t+T+1$ up to period $t+T+T$ appear. That is, in the model equations they are considering, expectations of variables outside their planning horizon show up. Forward-looking agents therefore need to give these expectations a value, without being able to solve in a sophisticated manner what will happen in these periods (since they lie outside their planning horizon).

Instead, they assume that in periods after their horizon, the model will have converged to a steady state, with all agents believing in steady state values for the future. All expectations about period $t+T+1$ up to period $t+T+T$ hence become zero. With this assumption, they are able to solve for period $t+T$ variables in terms of the state variables b_{t+T} and s_{t+T-1} (by solving a linear system of equations). They then move to the model equations of period $t+T-1$. For the expectations about $t+T$ variables of forward-looking agents, they plug in the (just calculated) solution of period $t+T$ variables. For expectations of backward-looking agents they use the expectations that backward-looking agents have currently formed about period $t+T$ variables. They then again assume steady state levels for expectations of variables outside their horizon. This allows them to solve for period $t+T-1$ variables in terms of state variable b_{t+T-1} and s_{t+T-2}.

This process goes on until they have solved for all expectations of forward-looking agents within their horizon in terms of the observed state variable b_t and s_{t-1}.

Expectations of variables for the periods within the horizon can then be obtained from these policy functions by plugging in the value of the current debt level.

D robustness

D.1 An expectations-driven liquidity trap triggered by a no-persistent preference shock.

Figure 8 shows impulse responses of the benchmark model with 50% backward-looking agents and $T = 8$ to a singe non-persistent negative preference shock. Comparing this figure with Figure 1 shows that such a one-time fundamental shock can trigger expectation driven liquidity traps very similar to those studied in the main body of the paper. The relative decrease in output versus inflation expectations depends on the exact nature of the shock, but this is not crucial for the occurrence of an expectation driven liquidity trap.

The only thing that is required is that inflation and our output gap expectations decrease enough for the zero lower bound to become binding and for backward-looking agents to keep on having pessimistic expectations for some periods. This could occur because of an explicitly modeled fundamental shock or because of a direct decrease of expectations due to factors that are not modeled explicitly, such as, sentiments, uncertainty and media announcements. The analysis in the main body of the paper allows for both interpretations, as is confirmed by this appendix.

D.2 Coefficient of mean reversion in expectations of backward-looking agents.

The length of liquidity traps not only depends on how many backward-looking agents there are in the economy and on their planning horizon, but also on how persistent these agents expect variables to be. Under the benchmark calibration backward-looking agents have an auto-regressive coefficient in their expectations of 0.8. When this coefficients is reduced, the following happens in a liquidity trap. Backward-looking agents expect faster

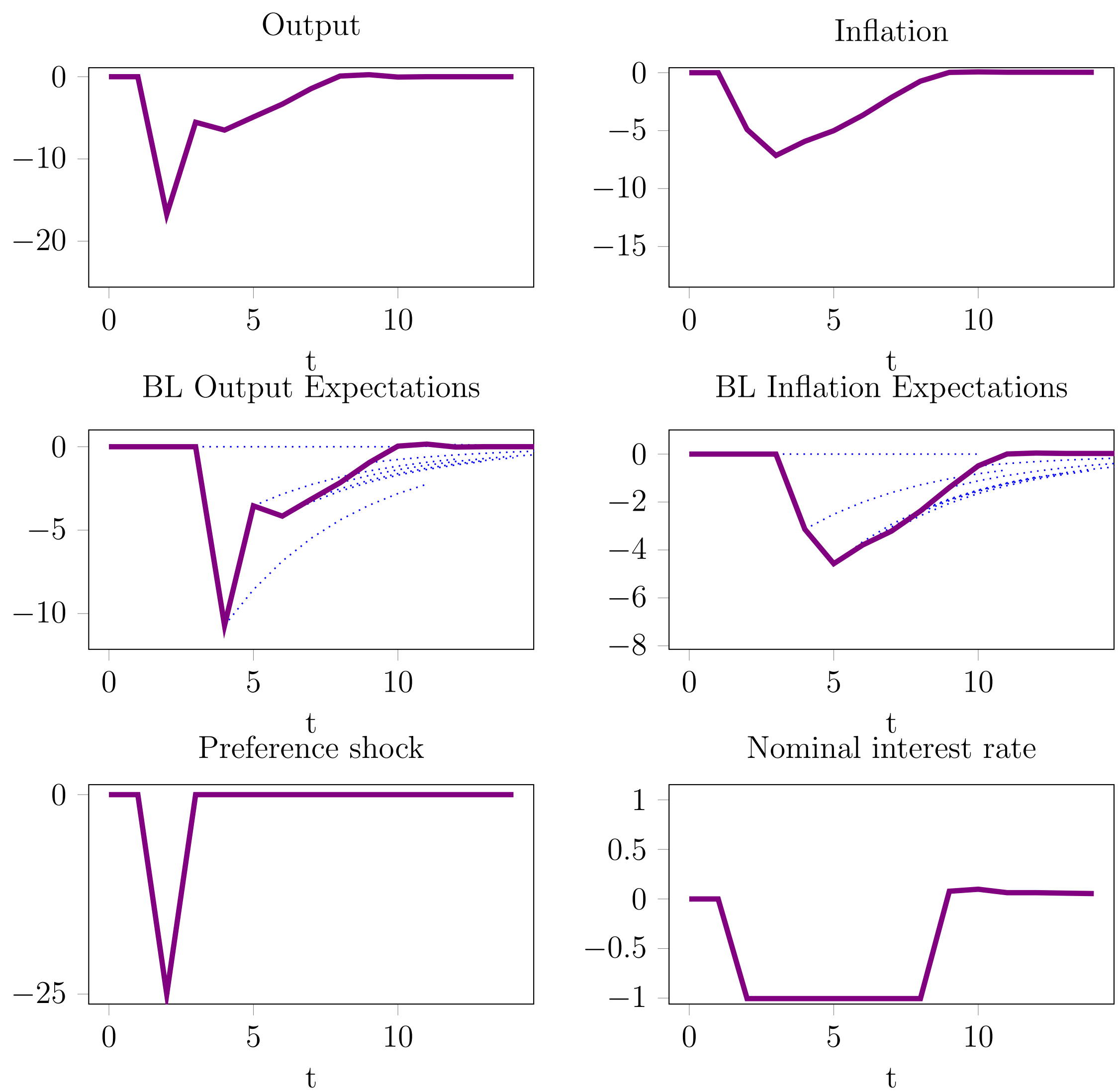


Figure 8: Liquidity trap for $T = 8$ after a single non-persistent shock with 50% backward-looking agents (expectations-driven liquidity trap triggered by a fundamental shock).

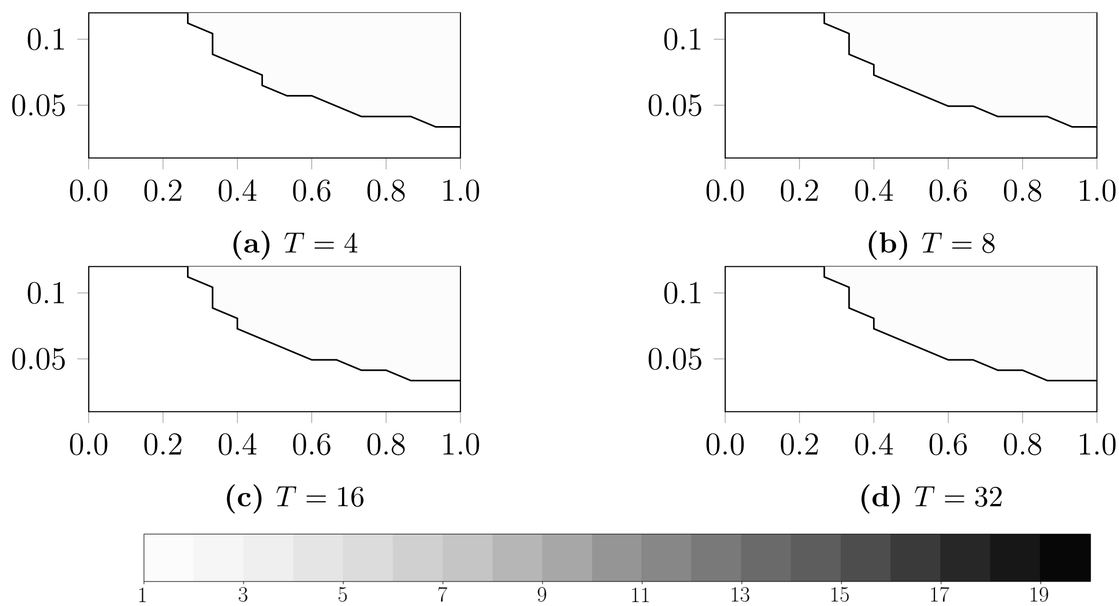
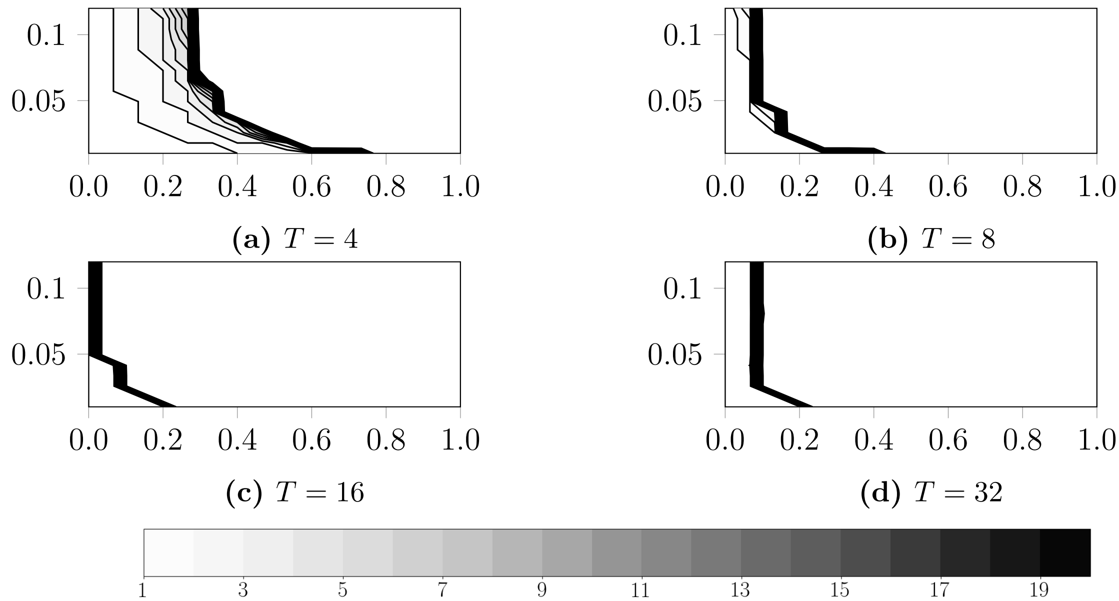


Figure 9: Length of liquidity trap in case of a lower auto-regressive coefficient in backward-looking expectations of $d = 0.5$ for different fractions of backward-looking agents (x-axis) and different sizes of the (non-persistent) negative shock to expectations (y-axis). The different panels correspond to different planning horizons. Darker color shades indicate a longer duration of the liquidity trap.

mean reversion towards the target steady state and expect the situation of low output and inflation not to last too long. This causes them to reduce consumption and prices less, so that inflation and output will indeed recover faster. This is illustrated in Figure 9, where the auto-regressive coefficient in expectations is reduced to 0.5. Liquidity traps now last at most 2 periods, even for large planning horizons. Deflationary spirals do not arise.

When on the other hand backward-looking agents expect lower mean reversion, they reduce prices and consumption more in a liquidity trap, causing the liquidity trap to last longer, and increasing the risk of falling in a deflationary spiral. Figure 10 depicts the extreme case of an auto-regressive coefficient in expectations of 1 so that in a liquidity trap backward-looking agents expect the liquidity trap to continue for all the periods within their planning horizon. Now deflationary spirals arise even when the fraction of backward-looking agents is not that large or when the planning horizon is small.



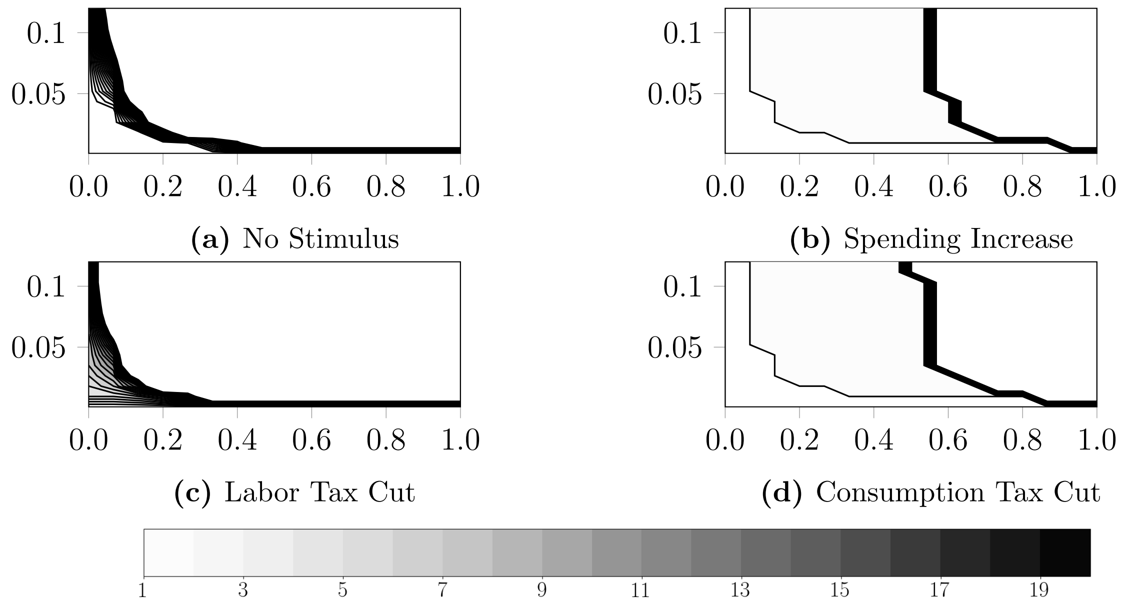
(a) $T = 4$ **(b)** $T = 8$ **(c)** $T = 16$ **(d)** $T = 32$

Figure 10: Length of liquidity trap in case of a higher auto-regressive coefficient in backward-looking expectations of $d = 1$ for different fractions of backward-looking agents (x-axis) and different sizes of the (non-persistent) negative shock to expectations (y-axis). The different panels correspond to different planning horizons. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas indicate liquidity traps of infinite length with ever decreasing inflation and output (deflationary spirals).

Note though, that the qualitative result that large planning horizons lead to longer liquidity traps and more deflationary spirals still holds, also for different values of the auto-regressive coefficient in expectations. However, when the the auto-regressive coefficient is reduced, differences between horizons become smaller, while they become more pronounced as the the auto-regressive coefficient in expectations is increased.

D.3 Adaptive learning

The main results of the paper qualitatively also hold when backward-looking agents are using a constant gain adaptive learning algorithm to form their expectations, as in, e.g., Benhabib et al. (2014). This is illustrated in Figure 11. This Figure reproduces the results of Figures 2. 5, 6 and 7 for a horizon of T=8 in case of steady state learning with constant



(a) No Stimulus

(b) Spending Increase

(c) Labor Tax Cut

(d) Consumption Tax Cut

Figure 11: Length of liquidity trap in the case where backward-looking agents us an adaptive learning algorithm with constant gain for $T = 8$. The x-axis corresponds to the fraction of backward0looking agents and the y-axis to the size of the (non-persistent) negative shock to expectations. Panel (a) depicts the case of no fiscal stimulus. Panels (b), (c) and (d) correspond to the cases of, respectively, spending increases, labor tax cuts and consumption tax cuts. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas indicate liquidity traps of infinite length with ever decreasing inflation and output (deflationary spirals).

gain. The gain parameter is here set to 0.05, which is within the range of values that is standard in the literature. Panel (a) of the figure corresponds to panel (b) of Figure 2; panel (b) of Figure 11 corresponds to panel (b) of Figure 5; and panel (c) and (d) correspond to panels (b) of, respectively, Figure 6 and Figure 7.

In panel (a) of Figure 11 it can be seen that longer liquidity traps and deflationary spirals arise as the fraction of backward-looking agents and the size of the shock to expectations become larger. Note, however, that deflationary spirals already occur for much smaller fractions of backward-looking agents than under the backward-looking expectations assumed in the main body of the paper. This can partly be explained by the fact that backward-looking agents now do not expect mean reversion to the target steady state, which leads to a similar situation as in Figure 10 in Appendix D.2.

For this reason, the required fiscal stimulus to prevent deflationary spirals is also larger than in the benchmark model. In panels (b) - (d) the assumed size of fiscal stimulus is 10 times the size of the shock. As can be seen in panels (b) and (d), deflationary spirals then still occur for large fractions of backward-looking agents.

However, as in the benchmark model, spending increases and consumption tax cuts can be highly effective in reducing the length of a liquidity trap and preventing a deflationary spiral. Labor tax cuts, on the other hand, (panel (c)) are not effective.

The qualitative effects of changing the planing horizon are also in line with the benchmark model. Results for different planning horizons under adaptive learning are available on request.

E Model under infinitely forward looking expectations

E.1 Output

When we let the planning horizon, T, go to infinity (71) can be written as

$$\hat{Y}_t = \frac{1}{\rho}\tilde{b}_t + g_t + \delta \sum_{s=0}^{\infty} \beta^s ((1-\bar{\tau}^l)((1-\alpha)E_t^F \hat{w}_{t+s} + \alpha E_t^b \hat{w}_{t+s}) - (1-\alpha)E_t^F \tilde{\tau}^l_{t+s} - \alpha E_t^b \tilde{\tau}^l_{t+s}) +$$

$$\frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho} \sum_{s=0}^{\infty} \beta^s ((1-\alpha)E_t^F \hat{\Xi}_{t+s} + \alpha E_t^b \hat{\Xi}_{t+s}) - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s ((1-\alpha)E_t^F \hat{LS}_{t+s} + \alpha E_t^b \hat{LS}_{t+s})$$

$$+ \frac{\mu\beta}{1-\beta}\xi_t - \mu \sum_{s=1}^{\infty} \beta^s ((1-\alpha)E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$- \mu_c \tilde{\tau}^c_t + \left(\frac{1}{\sigma} - 1\right) \frac{1-\bar{g}}{\rho}\bar{\Pi} \sum_{s=1}^{\infty} \beta^s ((1-\alpha)E_t^F \tilde{\tau}^c_{t+s} + \alpha E_t^b \tilde{\tau}^c_{t+s})$$

$$- \frac{\mu\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s (((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s+1} + \alpha E_t^b \hat{\pi}_{t+s+1}))$$

$$+ \frac{\bar{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s}))$$

Leading this equation 1 period and taking forward looking expectations gives

$$E_t^F \hat{Y}_{t+1} = \delta \sum_{s=1}^{\infty} \beta^{s-1} ((1-\bar{\tau}^l)((1-\alpha)E_t^F \hat{w}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{w}_{t+s}) - (1-\alpha)E_t^F \tilde{\tau}^l_{t+s} - \alpha E_t^F E_{t+1}^b \tilde{\tau}^l_{t+s})$$

$$+ \frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \hat{\Xi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\Xi}_{t+s}) - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s ((1-\alpha)E_t^F \hat{LS}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{LS}_{t+s})$$

$$+ E_t^F g_{t+1} + \frac{1}{\rho}\tilde{b}_{t+1} + \frac{\mu\beta}{1-\beta} E_t^F \hat{\xi}_{t+1} - \mu \sum_{s=2}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \hat{\xi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\xi}_{t+s})$$

$$- \mu_c E_t^F \tilde{\tau}^c_{t+1} + \left(\frac{1}{\sigma} - 1\right) \frac{1-\bar{g}}{\rho}\bar{\Pi} \sum_{s=2}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \tilde{\tau}^c_{t+s} + \alpha E_t^F E_{t+1}^b \tilde{\tau}^c_{t+s})$$

$$- \frac{\mu\beta}{1-\beta} \sum_{s=1}^{\infty} \beta^{s-1} (((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s+1} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s+1}))$$

$$+ \frac{\bar{b}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} (\beta((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s}))$$

Under the assumption (maintained throughout the main body of the paper) that $E_t^F E_{t+1}^b \hat{x}_{t+s} = E_t^b \hat{x}_{t+s}$ for $1 < s \leq T$, and plugging in backward-looking expectations and wages and profits, output can be written recursively as

$$
\begin{aligned}
&(1-\nu_y)\hat{Y}_t = (\beta-\alpha\beta\nu_y)E_t^f\hat{Y}_{t+1} + \alpha\beta d^2\nu_y\hat{Y}_{t-1} + \frac{1}{\rho}(\tilde{b}_t - \beta\tilde{b}_{t+1}) + (1+\nu_g)\tilde{g}_t - (\beta+\alpha\beta\nu_g)E_t^f\tilde{g}_{t+1} \\
&+ \alpha\beta d^2\nu_g\tilde{g}_{t-1} + \nu_\tau(\tilde{\tau}_t^l + \alpha\beta(d^2\tilde{\tau}_{t-1}^l - E_t^f\tilde{\tau}_{t+1}^l)) + \nu_s(\hat{s}_t + \alpha\beta(d^2\hat{s}_{t-1} - E_t^f\hat{s}_{t+1})) \\
&- \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}(\hat{LS}_t + \alpha\beta(d^2\hat{LS}_{t-1} - E_t^f\hat{LS}_{t+1})) + \frac{\mu\beta}{1-\beta}\hat{\xi}_t - \left(\beta\frac{\mu\beta}{1-\beta} + \mu\beta(1-\alpha)\right)E_t^f\hat{\xi}_{t+1} \\
&+ \nu_{c1}\tilde{\tau}_t^c + \nu_{c2}\alpha\beta d^2\tilde{\tau}_{t-1}^c + (-\beta\nu_{c1} + \beta(1-\alpha)\nu_{c2})E_t^f\tilde{\tau}_{t+1}^c \\
&+ (\frac{\mu\beta}{1-\beta}(1-\alpha) + \alpha\beta\frac{\bar{b}}{\rho})E_t^F\hat{\pi}_{t+1} + (\frac{\mu\beta}{1-\beta}\alpha d^2 - \alpha\beta\frac{\bar{b}}{\rho}d^2)\hat{\pi}_{t-1} \\
&- \frac{\bar{b}}{\rho}\hat{\pi}_t + (\frac{\bar{b}\beta}{\rho} - \frac{\mu\beta}{1-\beta})(\hat{i}_t + \alpha\beta(d^2\hat{i}_{t-1} - E_t^f\hat{i}_{t+1}))
\end{aligned}
$$

where I follow the assumption that all agents can observe contemporaneous variables when making their decision (but not yet when forming expectations), and the assumption that backward-looking agents do not anticipate future shocks.

E.2 Inflation

When T goes to infinity, (78) can be written as

$$
\begin{aligned}
\hat{\pi}_t =& \psi_t + \kappa_{y1}\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{Y}_{t+s} + \alpha E_t^b\hat{Y}_{t+s}) \\
&\kappa_s\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{s}_{t+s} + \alpha E_t^b\hat{s}_{t+s}) + \kappa_\tau\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\tilde{\tau}_{t+s} + \alpha E_t^b\tilde{\tau}_{t+s}) \\
&+ \frac{\kappa_{\pi 1}}{1-c_1}\sum_{s=1}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{\pi}_{t+s} + \alpha E_t^b\hat{\pi}_{t+s}) + \kappa_{\xi 1}\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{\xi}_{t+s} + \alpha E_t^b\hat{\xi}_{t+s})
\end{aligned}
\tag{111}
$$

with

$$\psi_t = \kappa_{y2} \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^b \hat{Y}_{t+s}) + \kappa_g \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \tilde{g}_{t+s} + \alpha E_t^b \tilde{g}_{t+s}) \tag{112}$$

$$+ \kappa_c \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^b \tilde{\tau}_{t+s}^c) + \kappa_{\xi 2} \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$+ \frac{\kappa_{\pi 2}}{1-c_2} \sum_{s=1}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s})$$

Writing one period ahead an taking forward-looking expectations gives

$$E_t^F \hat{\pi}_{t+1} = E_t^F \psi_{t+1} + \kappa_{y1} \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{Y}_{t+s}) \tag{113}$$

$$+ \kappa_s \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{s}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{s}_{t+s}) + \kappa_\tau \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \tilde{\tau}_{t+s} + \alpha E_t^F E_{t+1}^b \tilde{\tau}_{t+s})$$

$$+ \frac{\kappa_{\pi 1}}{1-c_1} \sum_{s=2}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s}) + \kappa_{\xi 1} \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$E_t^F \psi_{t+1} = \kappa_{y2} \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^b \hat{Y}_{t+s}) + \kappa_g \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \tilde{g}_{t+s} + \alpha E_t^b \tilde{g}_{t+s})$$

$$+ \kappa_c \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^b \tilde{\tau}_{t+s}^c) + \kappa_{\xi 2} \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$+ \frac{\kappa_{\pi 2}}{1-c_2} \sum_{s=2}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s}) \tag{114}$$

Plugging in expectations of backward-looking agents we can write

$$\begin{aligned}\hat{\pi}_t =& c_1 E_t^F \hat{\pi}_{t+1} + \psi_t - c_1 E_t^F \psi_{t+1} + \kappa_{y1}(\hat{Y}_t + \alpha c_1 (d^2 \hat{Y}_{t-1} - E_t^f \hat{Y}_{t+1})) \\ & \kappa_s(\hat{s}_t + \alpha c_1(d^2 \hat{s}_{t-1} - E_t^f \hat{s}_{t+1})) + \kappa_\tau(\tilde{\tau}_t^l + \alpha c_1(d^2 \tilde{\tau}_{t-1}^l - E_t^f \tilde{\tau}_{t+1}^l)) \\ & + \frac{c_1 \kappa_{\pi 1}}{1-c_1}((1-\alpha)E_t^F \hat{\pi}_{t+1} + \alpha E_t^b \hat{\pi}_{t+1}) + \kappa_{\xi 1}(\hat{\xi}_t - \alpha c_1 E_t^f \hat{\xi}_{t+1})\end{aligned} \tag{115}$$

and

$$\begin{aligned}\psi_t =& c_2 E_t^F \psi_{t+1} + \kappa_{y2}(\hat{Y}_t + \alpha c_2(d^2 \hat{Y}_{t-1} - E_t^f \hat{Y}_{t+1})) \\ & + \kappa_g(\tilde{g}_t + \alpha c_2(d^2 \tilde{g}_{t-1} - E_t^f \tilde{g}_{t+1})) + \kappa_c(\tilde{\tau}_t^c + \alpha c_2(d^2 \tilde{\tau}_{t-1}^c - E_t^f \tilde{\tau}_{t+1}^c)) \\ & + \frac{c_2 \kappa_{\pi 2}}{1-c_2}((1-\alpha)E_t^F \hat{\pi}_{t+1} + \alpha E_t^b \hat{\pi}_{t+1}) + \kappa_{\xi 2}(\hat{\xi}_t - \alpha c_2 E_t^f \hat{\xi}_{t+1})\end{aligned} \tag{116}$$

E.3 Fully rational expectations

Now, I consider the robustness of the assumption that forward-looking agents do not anticipate how backward-looking agents will revise their expectations in future periods. In case of an infinite planning horizon, forward-looking agents become fully rational when they take account of how backward-looking agents will revise their expectations.

Following similar steps as above, output and inflation can then be written recursively as

$$
\begin{aligned}
(1-z_1\nu_y)\hat{Y}_t &= (\beta-\alpha\beta\nu_y)E_t^f\hat{Y}_{t+1} + z_2\nu_y\hat{Y}_{t-1} + \frac{1}{\rho}(\tilde{b}_t-\beta\tilde{b}_{t+1}) + (1+z_1\nu_g)\tilde{g}_t - (\beta+\alpha\beta\nu_g)E_t^f\tilde{g}_{t+1} \\
&+ z_2\nu_g\tilde{g}_{t-1} + \nu_\tau(z_1\tilde{\tau}_t^l + z_2\tilde{\tau}_{t-1}^l - \alpha\beta E_t^f\tilde{\tau}_{t+1}^l) + \nu_s(z_1\hat{s}_t + z_2\hat{s}_{t-1} - \alpha\beta E_t^f\hat{s}_{t+1}) \\
&+ \frac{\mu\beta}{1-\beta}\hat{\xi}_t - \left(\beta\frac{\mu\beta}{1-\beta} + \mu\beta(1-\alpha)\right)E_t^f\hat{\xi}_{t+1} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}(z_1\hat{LS}_t + z_2\hat{LS}_{t-1} - \alpha\beta E_t^f\hat{LS}_{t+1}) \\
&+ \left(\nu_{c1} - \nu_{c2}\alpha\frac{\beta^2d^2}{1-\beta d}\right)\tilde{\tau}_t^c + \nu_{c2}z_2\tilde{\tau}_{t-1}^c + (-\beta\nu_{c1} + \beta(1-\alpha)\nu_{c2})E_t^f\tilde{\tau}_{t+1}^c \\
&+ \frac{\mu\beta}{1-\beta}((1-\alpha)E_t^F\hat{\pi}_{t+1} + \frac{\alpha d^2}{1-\beta d}\hat{\pi}_{t-1} - \frac{\alpha\beta d^2}{1-\beta d}\hat{\pi}_t) \\
&- \frac{\bar{b}}{\rho}(z_1\hat{\pi}_t + z_2\hat{\pi}_{t-1} - \alpha\beta E_t^f\hat{\pi}_{t+1}) + (\frac{\bar{b}\beta}{\rho} - \frac{\mu\beta}{1-\beta})(z_1\hat{i}_t + z_2\hat{i}_{t-1} - \alpha\beta E_t^f\hat{i}_{t+1}),
\end{aligned}
\tag{117}
$$

and

$$
\begin{aligned}
\hat{\pi}_t =& c_1E_t^F\hat{\pi}_{t+1} + \psi_t - c_1E_t^F\psi_{t+1} + \kappa_{y1}(z_3\hat{Y}_t + z_4\hat{Y}_{t-1} - \alpha c_1E_t^f\hat{Y}_{t+1}) \\
&\kappa_s(z_3\hat{s}_t + z_4\hat{s}_{t-1} - \alpha c_1E_t^f\hat{s}_{t+1}) + \kappa_\tau(z_3\tilde{\tau}_t^l + z_4\tilde{\tau}_{t-1}^l - \alpha c_1E_t^f\tilde{\tau}_{t+1}^l) \\
&+ \frac{c_1\kappa_{\pi1}}{1-c_1}((1-\alpha)E_t^F\hat{\pi}_{t+1} + \frac{\alpha d^2}{1-c_1d}\hat{\pi}_{t-1} - \frac{\alpha c_1d^2}{1-c_1d}\hat{\pi}_t) + \kappa_{\xi1}(\hat{\xi}_t - \alpha c_1E_t^f\hat{\xi}_{t+1})
\end{aligned}
\tag{118}
$$

With,

$$
\begin{aligned}
\psi_t =& c_2E_t^F\psi_{t+1} + \kappa_{y2}(z_5\hat{Y}_t + z_6\hat{Y}_{t-1} - \alpha c_2E_t^f\hat{Y}_{t+1}) \\
&+ \kappa_g(z_5\tilde{g}_t + z_6\tilde{g}_{t-1} - \alpha c_2E_t^f\tilde{g}_{t+1}) + \kappa_c(z_5\tilde{\tau}_t^c + z_6\tilde{\tau}_{t-1}^c - \alpha c_2E_t^f\tilde{\tau}_{t+1}^c) \\
&+ \frac{c_2\kappa_{\pi2}}{1-c_2}((1-\alpha)E_t^F\hat{\pi}_{t+1} + \frac{\alpha d^2}{1-c_2d}\hat{\pi}_{t-1} - \frac{\alpha c_2d^2}{1-c_2d}\hat{\pi}_t) + \kappa_{\xi2}(\hat{\xi}_t - \alpha c_2E_t^f\hat{\xi}_{t+1}),
\end{aligned}
\tag{119}
$$

$$
z_1 = 1 - \frac{\alpha\beta^2d^2}{1-\beta d} \tag{120}
$$

$$
z_2 = \frac{\alpha\beta d^2}{1-\beta d} \tag{121}
$$

$$z_3 = 1 - \frac{\alpha c_1^2 d^2}{1 - c_1 d} \tag{122}$$

$$z_4 = \frac{\alpha c_1 d^2}{1 - c_1 d} \tag{123}$$

$$z_5 = 1 - \frac{\alpha c_2^2 d^2}{1 - c_2 d} \tag{124}$$

$$z_6 = \frac{\alpha c_2 d^2}{1 - c_2 d} \tag{125}$$